



## Math Workshop College Algebra

**Workshop Objective:** Once a student successfully completes the Prealgebra Math Workshop, s/he will achieve a basic understanding the material taught in a MAT 140 course offered at Oakton. Students will be offered the skills and knowledge required of them to demonstrate competency on the Math Placement Test in any of the topics covered during the workshop.

**Workshop Topics:** Functions, Inverse Functions, Polynomial Functions, Fundamental Theorem of Algebra, Rational Functions, Conic Sections, Exponential and Logarithmic Functions and Equations, Systems of Linear and Non-linear Equations/Inequalities, Sequences and Series, Applications.

**Workshop Overview:** The workshop will identify and explain all of the various topics listed above. The topics covered during the workshop are found on the Oakton Math Placement Test. The supplementary exercises provided are a review of what a student would expect to learn in a MAT 140 course. **Any exercise sets labeled with an asterisk are additional (\*) topics not necessarily found on the Math Placement Test.**

Once a student feels confident with all of these workshop topics, then another, more advanced Math Workshop is recommended. A student may re-take any workshop. Each student has an opportunity to re-take the Math Placement Test one time, if they are not satisfied with their test performance. Please feel free to discuss any of your concerns with your Math Workshop instructor. This workshop is not intended to be a substitute for taking/completing MAT 140 over a normal semester setting. Command in the topic material, and supplementary exercises, is not a guarantee that a student has qualified to place into a higher math course (above MAT 140). However, if a student does have a solid understanding of the topics covered, then that student should place into a higher math course (above MAT 140).

# Functions

Find the domain and range of the following functions.

1)  $f(x) = x$  (linear function)

2)  $g(x) = x^2$  (square function)

3)  $h(x) = \sqrt{x}$  (square root function)

4)  $p(x) = 3x^3 - 5x^2 + 10x - 6$  (cubic/polynomial function of degree 3)

5)  $c(x) = 5$  (constant function)

6)  $r(x) = \frac{4x-5}{(x-5)(x-4)}$  (rational function)

Find the minimum and maximum values of the following functions on the given intervals.

7)  $f(x) = \sqrt{4-x}$  on  $-5, 4$

8)  $g(x) = \frac{2}{x-6}$  on  $7, 10$

9)  $h(x) = -4x^2$  on  $-2, 3$

# Operations with Functions

1) Given  $f(x) = x^2 - 4x + 3$  and  $g(x) = x - 1$ , find the following:

a)  $f(2) - g(-2)$

b)  $\frac{f(-1)}{g(1)}$

c)  $f(0) \cdot g(4)$

d)  $3 \cdot f(4) + 4 \cdot g(4)$

e)  $f[g(2)]$

f)  $g[f(-1)]$

g)  $f(2a)$

h)  $g(a - 2)$

2) Given  $f(x) = |x + 2| - 6$  and  $g(x) = x - 6$ , find the following:

a)  $f[g(1)]$

b)  $g[f(2)]$

c)  $f[g(-1)]$

d)  $g[f(-2)]$

e)  $f[g(b - 4)]$

f)  $g[f(3 - b)]$

## Inverse Functions

Find the inverse function of the following functions.

1)  $f(x) = 2x - 4$

2)  $g(x) = x^2 - 5, x \geq 0$

3)  $h(x) = |x - 2|, x \geq 2$

4)  $p(x) = x^3 - 1$

## Polynomial Functions

Graph the following. Identify all of the zeros/x-intercepts, y-intercept, and asymptotes.

Use a graphing calculator.

1)  $f(x) = 5(x - 5)(x + 2)(x - 1)$

2)  $g(x) = 4x^4 - 2x^3 - 24x^2 + 2x + 20$

3)  $p(x) = x^4 - x^3 - 19x^2 - x - 20$

4)  $h(x) = 8x^4 - 8x^3 + 26x^2 - 32x - 24$

Identify the rational roots of the following polynomials. Use the rational roots theorem.

5)  $p(x) = 2x^3 - x^2 - 43x + 60$

6)  $g(x) = 5x^3 + x^2 - 180x - 36$

7)  $h(x) = -70x^3 - 108x^2 + 270x - 108$

8)  $f(x) = 9x^4 - 18x^3 - x^2 + 18x - 8$

## Rational Functions

Graph the following. Identify all of the zeros/x-intercepts, y-intercept, and asymptotes.

Use a graphing calculator.

1)  $R(x) = \frac{x^2 - 16}{x^3 + 64}$

2)  $G(x) = \frac{2x - 6}{(x - 3)(x - 1)}$

3)  $B(x) = \frac{4x^3 - 4}{x^2 - 10}$

4)  $M(x) = \frac{x^3 - 10x^2 + 29x - 20}{x^3 + 4x^2 - 7x - 10}$

## Conic Sections

Graph the following.

- 1)  $9x^2 - 4y^2 = 36$  (hyperbola)
- 2)  $9y^2 - 16x^2 - 54y = 63$  (hyperbola)
- 3)  $(x-3)^2 + y^2 = 16$  (circle)
- 4)  $25x^2 + 4y^2 = 100$  (ellipse)
- 5)  $3x^2 + 6x + 3y^2 - 12y = 49$

## Exponential and Logarithmic Functions and Equations

Graph the following. Identify all intercepts and asymptotes.

- 1)  $t(x) = \ln[x]$
- 2)  $m(x) = \log[x]$
- 3)  $p(x) = e^x$
- 4)  $h(x) = 2^x$
- 5)  $g(x) = \ln[4-x] - 5$
- 6)  $f(x) = -e^{-(x-4)} + 3$
- 7)  $s(x) = \left(\frac{1}{2}\right)^x - 1$
- 8)  $n(x) = \log_{\frac{1}{2}} x + 2$

Solve.

- 9)  $\log_3 x = 27$
- 10)  $\log_2 x + \log_2(x-2) = 3$
- 11)  $\log_4(x+5) = 3$
- 12)  $\log(50x) - \log(x-5) = 2$
- 13)  $\log[x+21] + \log[x] = 2$
- 14)  $8 + \ln \sqrt{x-5} = 10$
- 15)  $3^x = 9$
- 16)  $4^x = 20$
- 17)  $100e^{0.01x} = 300$
- 18)  $5x \ln 2 = \ln 5$

$$19) \log_x \frac{8}{27} = 3$$

$$20) 3^{2x-1} = 4^{x+2}$$

$$21) e^{x^2} = 200$$

$$22) \log_3 (3m^2)^{\frac{1}{4}} - 1 = 2$$

$$23) \log_a (x+6) - \log_a (x+2) = \log_a x$$

$$24) \frac{e^x - 1}{e^{-x} - 1} = -3$$

## Systems of Linear and Non-linear Equations/Inequalities

Solve.

$$1) 2x - 4y = 8$$

$$x + y = 4$$

$$2) y = 2x - 6$$

$$12x - 6y = 9$$

$$3) x + y + z = 1$$

$$2x - y + 2z = 6$$

$$4x - y - z = 10$$

$$4) 4a - b = 6$$

$$c - b = 10$$

$$2a - 3c + 5b = 8$$

Solve.

$$5) 2x - y = 6$$

$$y = x^2$$

$$6) y = x - 3$$

$$x^2 + y^2 = 4$$

$$7) xy = 1$$

$$x + y = 2$$

$$8) y = 2x^2 + 3x + 4$$

$$y = x^2 + 2x + 3$$

$$9) y = x^2$$

$$x^2 + (y - 2)^2 = 1$$

Graph the solution.

10)  $x - y < 4$

$$2x + 6y \geq 10$$

11)  $4x + 8y \leq 0$

$$y < 4 + x$$

## Rational Inequalities

Solve and graph.

1)  $\frac{x-1}{(x+2)(x-3)} > 0$

2)  $\frac{x-2}{4x} \geq 1$

3)  $\frac{x+1}{x-3} \leq \frac{x-2}{x+3}$

## Sequences and Series

Write the terms of the arithmetic sequences satisfying each of the following conditions.

1)  $a_1 = 5, d = -2, n = 6$

2)  $a_1 = -5, a_2 = -5 + \sqrt{3}, n = 4$

Find the formula for the following arithmetic sequences and the next two elements.

3) 6, 8, 10, 12, 14

4) -4, -7, -10, -13, -16

5) 3, 8, 15, 24, 35

\*Find  $a_1$  for each of the following arithmetic sequences.

6)  $a_5 = 27, a_{15} = 87$

7)  $S_{16} = -160, a_{16} = -25$

Write the terms of the geometric sequences satisfying the following conditions.

8)  $a_1 = \frac{5}{3}, r = 3, n = 4$

9)  $a_4 = 5, a_5 = 10, n = 5$

Find the formula for the following geometric sequences and the next two elements.

10)  $\left\{ \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \frac{32}{243}, \frac{64}{729} \right\}$

11)  $\left\{ \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64} \right\}$

Find  $a_5$  and  $a_n$  for each of the following geometric sequences.

12)  $a_2 = -4, r = 3$

13)  $\frac{4}{5}, 2, 5, \frac{25}{2}$

Find the next element for each sequence. Find the formula for each sequence.

14) 5, 10, 17, 26, 37 (recursive)

15)  $\left\{ \frac{16}{15}, \frac{7}{5}, \frac{26}{15}, \frac{31}{15}, \frac{12}{5} \right\}$  (geometric)

\*Find the following.

16)  $\sum_{k=0}^{10} k$

17)  $\sum_{i=1}^5 1$

18)  $\sum_{n=1}^{80} 2n - 5$

19)  $\sum_{k=1}^{50} k^2 - k + 3$

20)  $\sum_{i=1}^{30} i^3$

Evaluate.

21)  $(4!)(5!)$

22)  $(3!)^2$

\*Find the following binomial expansions.

23)  $(x+1)^7$

24)  $(3x-2)^4$

\*Find the  $k^{\text{th}}$  term of the following binomial expansions.

25) Twelfth term of  $(2x + y^2)^{16}$

26) Tenth term of  $(x^3 + 3y)^{11}$

# Applications

1) If  $r = 3t + 6$ , and  $t = 6 - 5s$ , then find the formula to express  $r$  in terms of  $s$ .

2) If  $a^{x^2} \left( \frac{1}{a^{10x}} \right) = \frac{1}{a^{25}}$ , then  $x = ?$

3) If  $2^{x^2} \cdot 2^{-7x} = \frac{1}{64}$ , then  $x = ?$

4) If the range of  $f(x) = x^2 - 6$  is all real numbers from 10 to 30, what positive numbers lie in the domain of  $f(x)$ ?

5) Given the following relation between Watts( $W$ ) and Decibels( $d$ ),

find the logarithmic function $d(W)$ .	<i>Watts(W)</i>	<i>Decibels(d)</i>
	200	210
	20	205
	2	200
	0.2	195
	0.02	190

6) The operation  $\square$  is defined to be  $x \square y = 3x + 5y$ . If  $5 \square x = 10$ , then  $x = ?$

7) The first term in an arithmetic series is 4 and the last term is 494, and the sum is 2739, what are the first 3 terms.

8) The sum of the first 17 terms of an arithmetic sequence is 187. If  $a_{17} = -13$ , find  $a_1$  and  $d$ .

9) For the functions,  $f(x)$  and  $g(x)$ , tables of values are given as follows:

<u><math>x</math></u>	<u><math>f(x)</math></u>	<u><math>x</math></u>	<u><math>g(x)</math></u>
0	-4	-1	5
1	5	5	10

Find  $g[f(1)]$ .

10) Given  $A = \begin{bmatrix} 3 & 4 \\ 6 & -5 \end{bmatrix}$  and  $B = \begin{bmatrix} -1 & 0 \\ -4 & 7 \end{bmatrix}$ , find  $A + B$ .

11) Patrick won \$2,000,000 in the state lottery. He first paid income tax on 20% of the winnings. Of the rest, he invested some at 8.5% and some at 7%, earning \$120,000. How much did he invest at each rate?

12) What is the length of the side of a square, if its area and perimeter are equal?

- 13) The height of a launched object is given by the position function defined by,  $s(t) = -16t^2 + v_0t + s_0$ , where  $t$  is the number of seconds after the object is launched,  $v_0$  is the initial velocity, and  $s_0$  is the initial height.
- A rocket is launched directly upward from ground level with an initial velocity of 100 feet per second.
- After how many seconds does the rocket reach its maximum height?
  - What is the maximum height reached by the rocket?
- 14) A quadratic equation  $g(x) = 0$  has a solution  $x = -4$ . Its graph has vertex  $(-2, 4)$ . Find the second solution to the equation.
- 15) Given  $f(x) = \log_3 x$ , find the following:
- $f(3^4)$
  - $f(3^{\log_2 4})$
  - $f(3^{3\log_5 125})$
- 16) Find the center and radius of the following circles:
- $x^2 + y^2 + 8x - 6y - 15 = 0$
  - $x^2 + y^2 + 10y - 75 = 0$

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## College Algebra Math Workshop Solutions

### Functions

1)  $D: \mathbb{R}, R: \mathbb{R}$

2)  $D: \mathbb{R}, R: [0, \infty)$

3)  $D: [0, \infty), R: [0, \infty)$

4)  $D: \mathbb{R}, R: \mathbb{R}$

5)  $D: \mathbb{R}, R: \{5\}$

6)  $D: \mathbb{R} - \{5, 4\}, R: \mathbb{R}$

7) min: 0, max: 3

8) min:  $\frac{1}{2}$ , max: 2

9) max: 0, min: -36

### Operations with Functions

1) a) 2

b) *undefined*

c) 9

d) 21

e) 0

f) 7

g)  $4a^2 - 8a + 3$

h)  $a - 3$

2) a) -3

b) -8

c) -1

d) -12

e)  $|b - 8| - 6$

f)  $|5 - b| - 12$

### Inverse Functions

1)  $f^{-1}(x) = \frac{x+4}{2}$

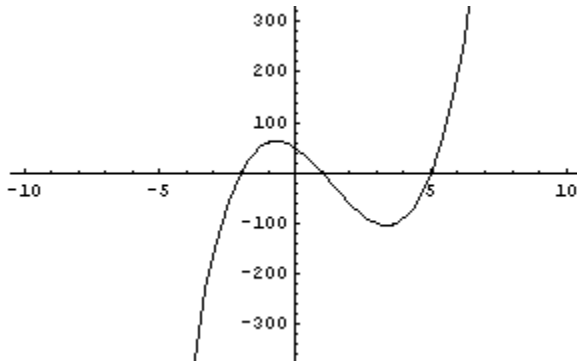
2)  $g^{-1}(x) = \sqrt{x+5}, x \geq -5$

3)  $h^{-1}(x) = x + 2, x \geq 0$

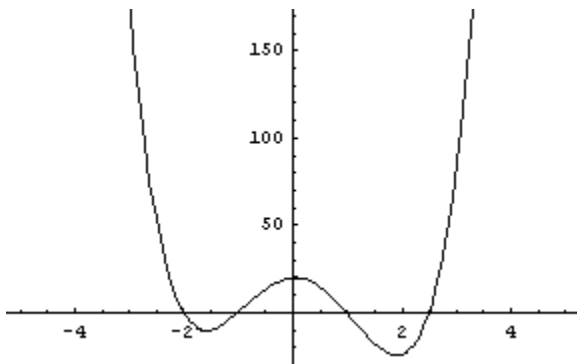
4)  $p^{-1}(x) = \sqrt[3]{x+1}$

## Polynomial Functions

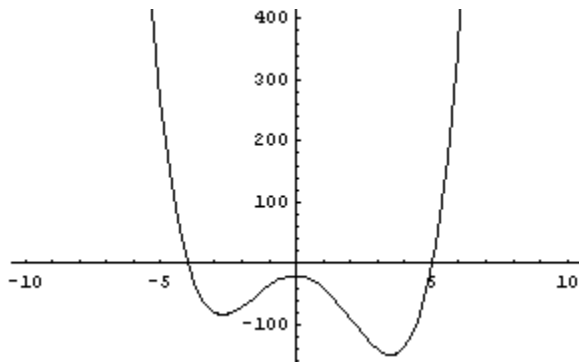
1) x-intercepts:  $5, -2, 1$  , y-intercept:  $50$



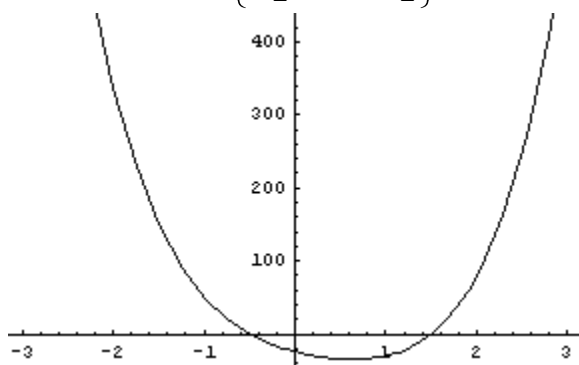
2) x-intercepts:  $\left\{-2, -1, \frac{5}{2}\right\}$  , y-intercept:  $20$



3) x-intercepts:  $-4, -i, i, 5$  , y-intercept:  $-20$



4) x-intercepts:  $\left\{-\frac{1}{2}, -2i, 2i, \frac{3}{2}\right\}$ , y-intercept:  $-24$



5)  $\left\{\frac{3}{2}, 4, -5\right\}$

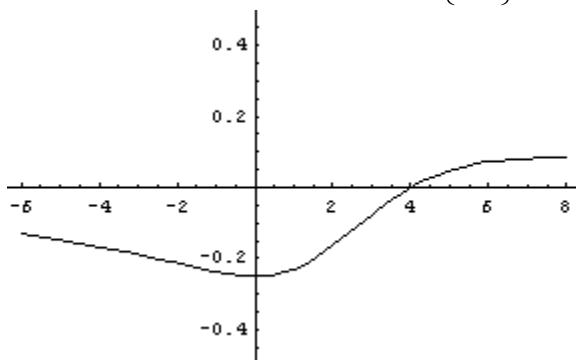
6)  $\left\{-\frac{1}{5}, 6, -6\right\}$

7)  $\left\{-3, \frac{6}{7}, \frac{3}{5}\right\}$

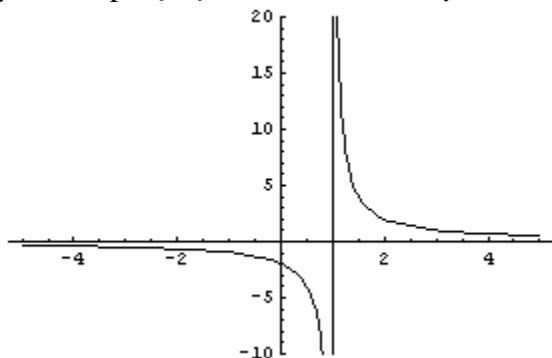
8)  $\left\{\frac{4}{3}, \frac{2}{3}, 1, -1\right\}$

#### Rational Functions

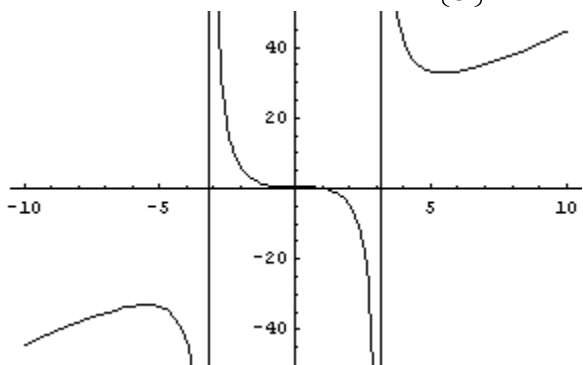
1) x-intercept:  $4$ , y-intercept:  $\left\{-\frac{1}{4}\right\}$ , VA:  $x = -1$ , HA:  $y = 0$



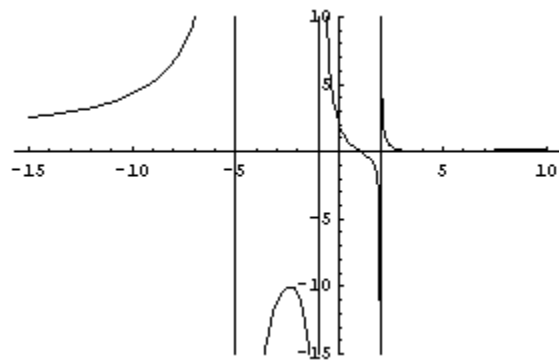
2) y-intercept:  $\{-2\}$ , VA:  $x = 1$ , HA:  $y = 0$



3) x-intercept: {1}, y-intercept:  $\left\{\frac{2}{5}\right\}$ , VA:  $x = \pm\sqrt{10}$ , OA:  $y = 4x$

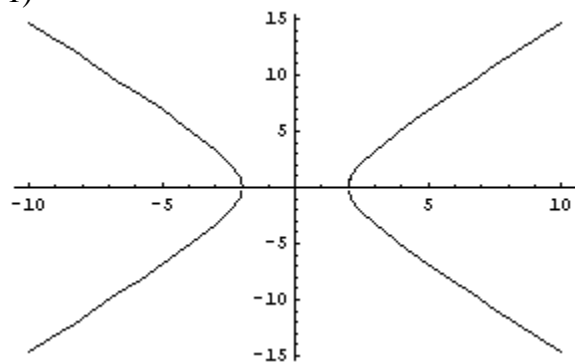


4) x-intercept: 1,4,5 , y-intercept: {2}, VA:  $-5,-1,2$  , HA:  $y = 1$

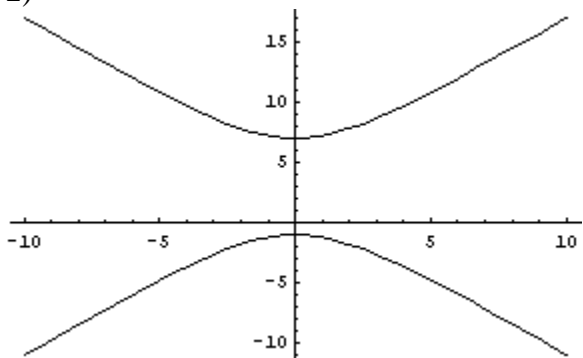


### Conic Sections

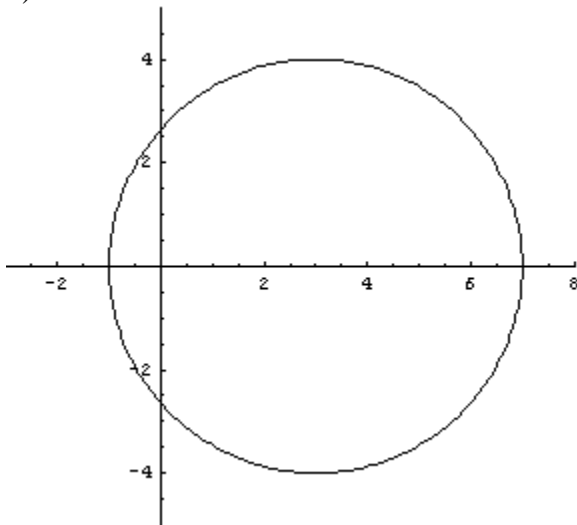
1)



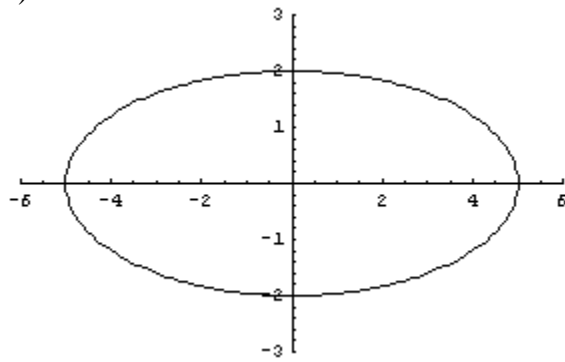
2)



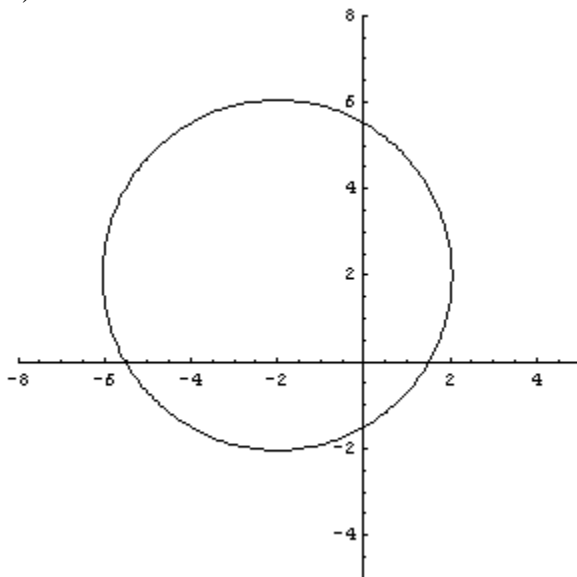
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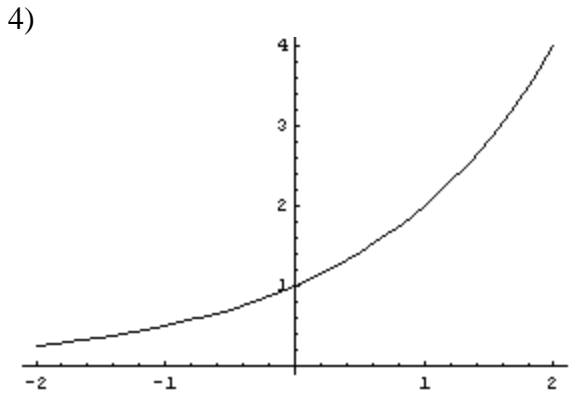
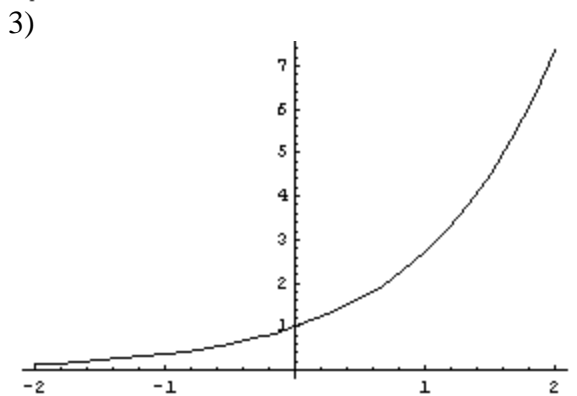
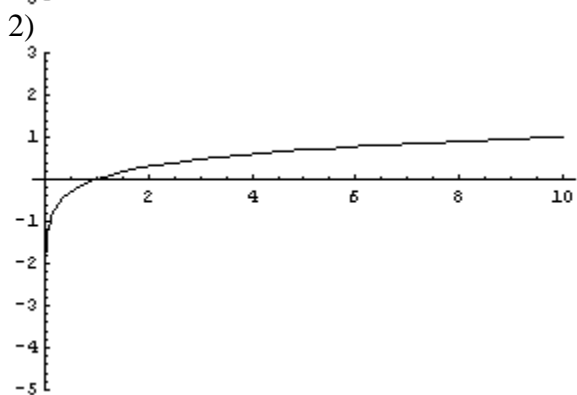
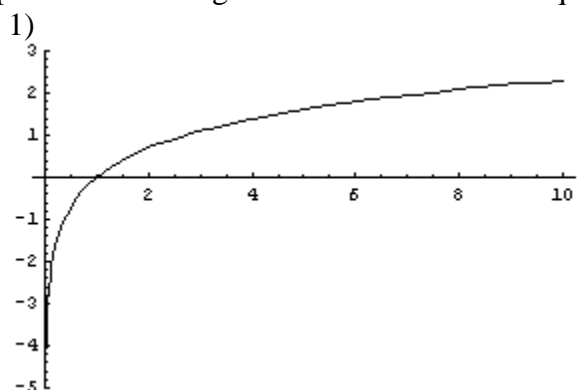
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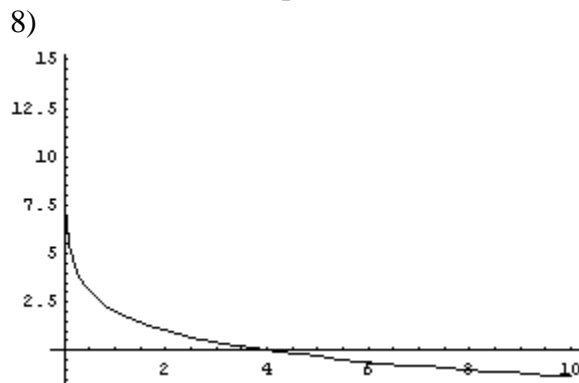
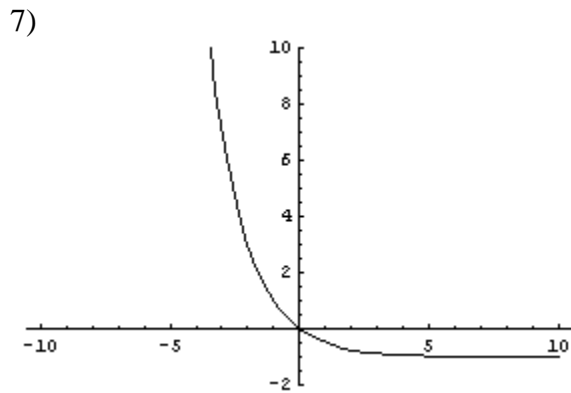
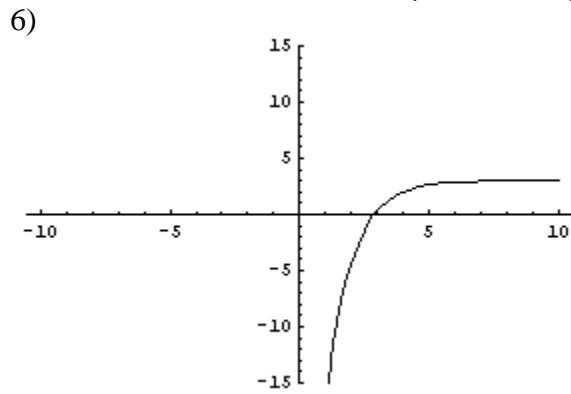
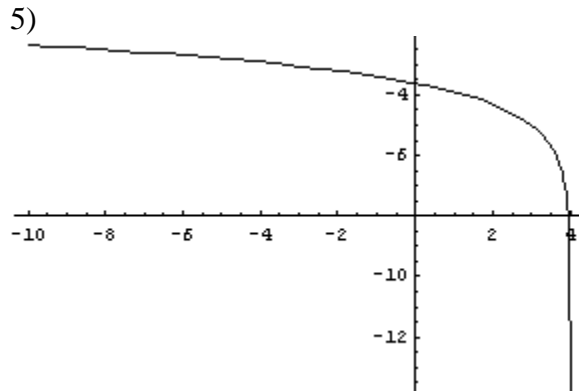


5)



# Exponential and Logarithmic Functions and Equations





9)  $x = 3^{27}$

10)  $x = 4$

11)  $x = 59$

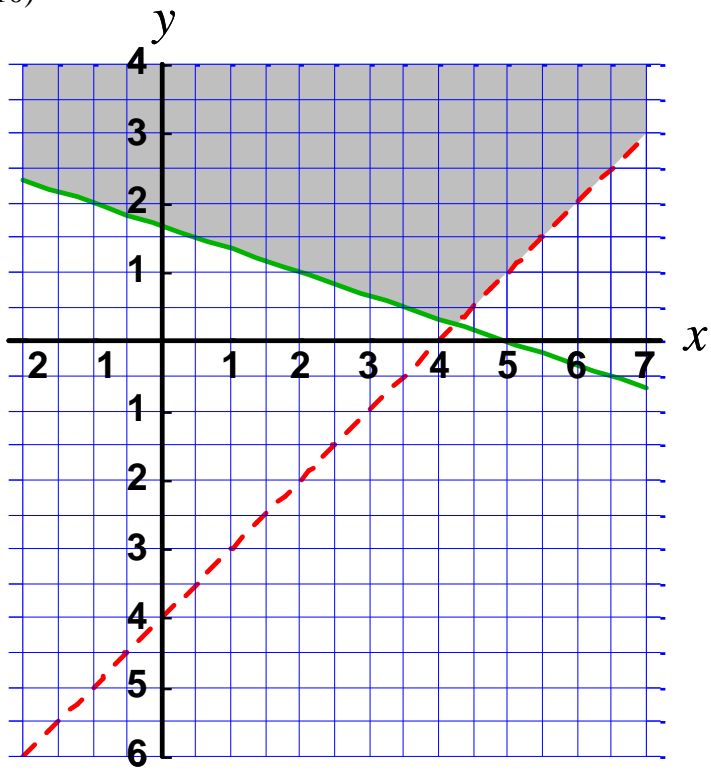
12)  $x = 10$

- 13)  $x = 4$
- 14)  $x = e^4 + 5$
- 15)  $x = 2$
- 16)  $x = \log_4 20$
- 17)  $x = \frac{\ln 3}{0.01}$
- 18)  $x = \frac{\ln 5}{\ln 32}$
- 19)  $x = \frac{2}{3}$
- 20)  $x = \frac{2 \log_3 4 + 1}{2 - \log_3 4}$  or  $x = \frac{2 \ln 4 + \ln 3}{2 \ln 3 - \ln 4}$
- 21)  $x = \pm \sqrt{\ln 200}$
- 22)  $m = 3^2$
- 23)  $x = 2$ , or  $x = -3$
- 24)  $x = \ln 3$

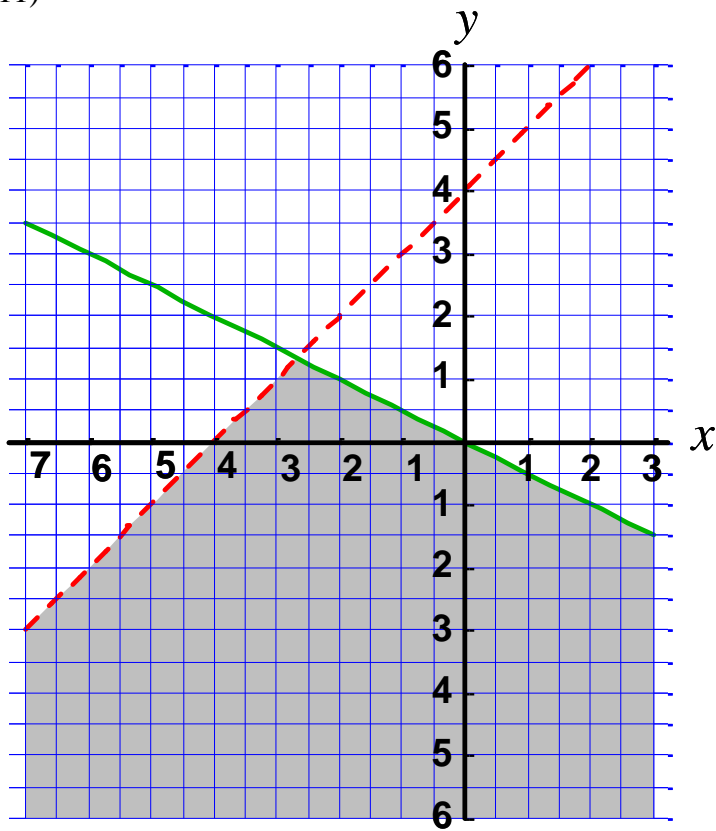
#### Systems of Linear and Non-linear Equations/Inequalities

- 1) 4,0
- 2) No solution
- 3)  $\left(\frac{11}{5}, -\frac{4}{3}, \frac{2}{15}\right)$
- 4) 5,14,24
- 5)  $1 - \sqrt{5}i, 2 - 2 - \sqrt{5}i$  ,  $1 + \sqrt{5}i, 2 - 2 + \sqrt{5}i$
- 6)  $\left(\frac{3}{2} - \frac{1}{2}i, -\frac{3}{2} - \frac{1}{2}i\right)$ ,  $\left(\frac{3}{2} + \frac{1}{2}i, -\frac{3}{2} + \frac{1}{2}i\right)$
- 7) 1,1
- 8)  $\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{3}{2} + \frac{\sqrt{3}}{2}i\right)$ ,  $\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{3}{2} - \frac{\sqrt{3}}{2}i\right)$
- 9)  $\left(\sqrt{\frac{5}{3}}, \frac{5}{3}\right)$ ,  $\left(\sqrt{\frac{5}{3}}, \frac{5}{3}\right)$

10)



11)



### Rational Inequalities

1)  $-2 < x < 1 \cup x > 3$

2)  $x \leq 1 - \sqrt{5} \cup x \geq 1 + \sqrt{5}$

3)  $x < -3 \cup \frac{1}{3} \leq x < 3$

### Sequences and Series

1)  $5, 3, 1, -1, -3, -5$

2)  $-5, -5 + \sqrt{3}, -5 + 2\sqrt{3}, -5 + 3\sqrt{3}$

3)  $a_n = 6 + 2n - 1$

4)  $a_n = -4 + -3n - 1$

5)  $a_n = 3 + 5n - 1$

6)  $a_1 = 3$

7)  $a_1 = 5$

8)  $\left\{ \frac{5}{3}, 5, 15, 45 \right\}$

9)  $\left\{ \frac{5}{8}, \frac{5}{4}, \frac{5}{2}, 5, 10 \right\}$

10)  $a_n = \frac{4}{9} \left( \frac{2}{3} \right)^{n-1}$

11)  $a_n = \frac{1}{4} \left( \frac{1}{2} \right)^{n-1}$

12)  $a_5 = -108, a_n = \left( -\frac{4}{3} \right) 3^{n-1}$  or  $a_n = -4 \cdot 3^{n-2}$

13)  $a_5 = \frac{125}{4}, a_n = \left( \frac{4}{5} \right) \left( \frac{5}{2} \right)^{n-1}$  or  $a_n = \frac{5^{n-2}}{2^{n-3}}$

14) 50

15)  $\frac{41}{15}$

16) 55

17) 5

18) 6080

- 19) 41800
- 20) 216225
- 21) 2280
- 22) 36
- 23)  $x^7 + 7x^6 + 21x^5 + 35x^4 + 35x^3 + 21x^2 + 7x + 1$
- 24)  $81x^4 - 216x^3 + 216x^2 - 96x + 16$
- 25)  $139,776x^5y^{22}$
- 26)  $55 \cdot 3^9 x^6 y^9$

Applications

- 1)  $r = -15s + 24$
- 2)  $x = 5$
- 3)  $x = 6$  or  $x = 1$
- 4) 4, 6
- 5)  $y = 200 + 5 \log\left(\frac{x}{2}\right)$
- 6)  $x = -1$
- 7)  $a_1 = 4, a_2 = 53, a_3 = 102$
- 8)  $a_1 = 35, d = -3$
- 9) 10
- 10)  $\begin{bmatrix} -19 & 28 \\ 14 & -35 \end{bmatrix}$
- 11) \$1,066,666.67 at 7% and \$533,333.33 at 8.5%
- 12) 4 units
- 13) a)  $t = 3.125$  sec b) 156.25 ft
- 14)  $x = 0$
- 15) a) 4 b) 2 c) 9
- 16) a) Center  $(-4, 3), r = 2\sqrt{10}$   
 b) Center  $(0, -5), r = 10$