



## **Math Workshop Intermediate Algebra**

**Workshop Objective:** Once a student successfully completes the Prealgebra Math Workshop, s/he will achieve a basic understanding the material taught in a MAT 120 course offered at Oakton. Students will be offered the skills and knowledge required of them to demonstrate competency on the Math Placement Test in any of the topics covered during the workshop.

**Workshop Topics:** Order of Operations with Real Numbers, Solve and Graph Linear Equations/Inequalities in One Variable, Absolute Value Equations/Inequalities, Compound Inequalities, Distance Formula, Midpoint Formula, Concepts of Lines, Solve and Graph Linear Inequalities in Two Variables, Graphing Circles, Solve and Graph Systems of Linear/Non-Linear Equations/Inequalities in Two Variables, Basic Operations with Polynomials, Factor Polynomials, Basic Operations with Rational Expressions, Variation, Exponents, Radical Expressions, Complex Numbers, Quadratic Equations/Inequalities, Formulas, Functions, Applications.

**Workshop Overview:** The workshop will identify and explain all of the various topics listed above. The topics covered during the workshop are found on the Oakton Math Placement Test. The supplementary exercises provided are a review of what a student would expect to learn in a MAT 120 course. **Any exercise sets labeled with an asterisk (\*) are additional topics not necessarily found on the Math Placement Test.**

Once a student feels confident with all of these workshop topics, then another, more advanced Math Workshop is recommended. A student may re-take any workshop. Each student has an opportunity to re-take the Math Placement Test one time, if they are not satisfied with their test performance. Please feel free to discuss any of your concerns with your Math Workshop instructor. This workshop is not intended to be a substitute for taking/completing MAT 120 over a normal semester setting. Command in the topic material, and supplementary exercises, is not a guarantee that a student has qualified to place into a higher math course (above MAT 120). However, if a student does have a solid understanding of the topics covered, then that student should place into a higher math course (above MAT 120).

## Order of Operations with Real Numbers

Simplify

$$1) \frac{1}{3} - \frac{2}{5} - \left(-\frac{4}{7}\right)$$

$$2) \left(1\frac{4}{5}\right) \times \left(2\frac{3}{4}\right)$$

$$3) -4^2 - 6^3 + (-5)^2$$

$$4) 120 \div (-2) \times (-6)$$

$$5) 4 - 3[-3^2 - 6 \div (-2) - 16 \div (-1)]^2$$

$$6) (0.045) \times (-0.20) \div (0.03)^2$$

$$7) \left(\frac{2}{7}\right) \left[\frac{1}{4} + \frac{1}{6}\right]$$

$$8) \left(\frac{1}{2}\right)^2 + \left(9 - \frac{1}{2}\right) \div -2$$

$$9) |-3^2 - 6^2| - |8 - 16|$$

$$10) \left(\frac{8}{3} \div \frac{2}{3}\right)^3 - \frac{1}{2}$$

## Solve and Graph Linear Equations/Inequalities in One Variable

Solve

$$1) 2x - 5 = -16$$

$$2) -x - 10 = 15$$

$$3) 4x - 4 = 2x + 16$$

$$4) 5(-2x - 10) - 10 = 2(-4x - 16)$$

$$5) -3 - 5(5x - 6) = \frac{2}{3}(x - 6) - \frac{4}{3}x$$

$$6) \frac{1}{3}x = -\frac{2}{6}$$

$$7) x + \left(-\frac{2}{3}\right) = \frac{5}{7}$$

Solve and Graph

8)  $x - 6 < 8$

9)  $-5x > -25$

10)  $-6x + 6 < 10$

11)  $2(x - 4) + 2x \geq -4x - 8$

12)  $-\frac{2}{3}x < 6$

13)  $\frac{1}{6}x + 3 \leq 2$

## Compound Inequalities

Simplify. U means "or", I means "and".

1)  $x < 5$  U  $x > -4$

2)  $x > -4$  U  $x > -10$

3)  $x \leq 10$  U  $x \leq 20$

4)  $x < 7$  I  $x \geq 3$

5)  $x < 6$  I  $x > 9$

Solve and Graph.

6)  $-6 < 2x - 6 < 8$

7)  $4 \leq -3x + 1 < 12$

## Absolute Value Equations/Inequalities

Solve

1)  $|x| = -5$

2)  $|2x - 5| = 5$

3)  $|3 - x| = |4x - 6|$

4)  $|x - 3| + 6 = 8$

5)  $|3x| - 1 = 8$

Solve and Graph

6)  $|x| < 4$

7)  $|x - 4| > 6$

8)  $|3x - 6| \leq 4$

9)  $-4|-2x - 6| \leq -4$

## Distance Formula

Find the distance between the following pairs of points.

- 1)  $(0,4)$  and  $(-3,9)$
- 2)  $(4,4)$  and  $(5,-6)$

## Midpoint Formula

Find the midpoint between the following pairs of points.

- 1)  $(0,5)$  and  $(0,-6)$
- 2)  $(4,-5)$  and  $(-3,-2)$

## Concepts of Lines

Given the slope and point of a line, find the equation of the following lines in slope-intercept form.

- 1)  $m = -\frac{2}{3}$ ,  $P(-3,4)$
- 2)  $m = 3$ ,  $Q(2,-6)$

Given two points of a line, find the equation of the following lines in slope-intercept form.

- 3)  $(-1,-4)$  and  $(0,5)$
- 4)  $(-4,5)$  and  $(-1,-1)$

Find a line parallel and perpendicular to the given lines that pass through the given point.

- 5)  $2x - 5y = 6$  ;  $(1,3)$
- 6)  $4x - 6y = 9$  ;  $(-1,4)$

## Solve and Graph Linear Inequalities in Two Variables

Solve and Graph.

- 1)  $2x - 6y < 12$
- 2)  $-3x + y \geq 9$

# Solve and Graph Systems of Linear Equations/Inequalities in Two Variables

Solve using Substitution and Elimination Methods.

1)  $x - y = 5$

$$2x + 5y = 10$$

2)  $3x - 7y = 15$

$$4x + 7y = 10$$

3)  $\frac{2}{3}x + \frac{1}{7}y = -11$

$$\frac{1}{7}x - \frac{1}{3}y = -10$$

4)  $0.7x - 0.3y = 0.5$

$$-0.4x + 0.7y = 1.3$$

5)  $4x - 8y = 12$

$$-3x - 9y = 12$$

Solve

6)  $6x - 4y + 5z = 31$

$$5x + 2y + 2z = 13$$

$$x + y + z = 2$$

7)  $6x - y - z = 12$

$$2x + y + z = -2$$

$$6x - 3y - 2z = 3$$

8)  $4a - 5b = 14$

$$a + b + c = 20$$

$$4c - 6b = 12$$

9)  $2x - 4y + 4z = 22$

$$-2x - y = -12$$

$$x - z = 10$$

Solve and Graph

10)  $y \leq x + 2$

$$x + 3y > 3$$

11)  $x \leq 2$

$$y \geq -5$$

12)  $2x + 4y > 8$

$$y < x - 6$$

## Basic Operations with Polynomials

Simplify

1)  $(-3x^2y^2 - 4x^2y + 5xy^2 + 10xy) + (-10x^2y^2 - 8x^2y - 6xy^2 - 4xy)$

2)  $(-5x^2 + 10x - 15) - (-6x^2 + 10x - 20)$

3)  $(3x - 5)^2$

4)  $(x - 3)^3$

5)  $(2x - 5)(2x + 5)$

6)  $(3x + 10)(6x - 5)$

7)  $(x - 1)(x^2 + x + 1)$

8)  $\left(x + \frac{2}{3}\right)\left(x - \frac{4}{5}\right)$

9)  $(y^3 + 125) \div (y + 5)$

10)  $(x^2 + 10x - 20) \div (x + 5)$

11)  $(x^3 - 4x + 12) \div (x - 2)$

12)  $(2x^5 - x^3 - 5x^2 + x - 6) \div (x^2 - 4)$

## Factor Polynomials

Factor

1)  $x^2 + 5x + 6$

2)  $2x^2 - 12x + 5x - 30$

3)  $16x^2y^2 - 1$

4)  $y^4 - 1$

5)  $25x^2 - 30x + 9$

6)  $12x^2 - 30x - 2x + 5$

7)  $21x^2 - 34x - 35$

8)  $5x^2 - 40x + 35$

9)  $4x^3 + 8x^2 + 4x$

10)  $x^3 + 1$

11)  $x^3 - 1$

12)  $64x^3y^3 - 8$

13)  $27x^6 + y^6$

# Basic Operations with Rational Expressions

Simplify

$$1) \frac{3}{x+3} + \frac{x}{x+3}$$

$$2) \frac{4x}{x-1} - \frac{2x}{x-1}$$

$$3) \frac{5x+1}{1-x} - \frac{x+1}{x-1}$$

$$4) \frac{x-2}{x+4} + \frac{x+3}{x-5}$$

$$5) \frac{-4}{x+2} + \frac{5}{x-2} + \frac{x+3}{x^2-4}$$

$$6) \frac{2xy}{x^2-y^2} + \frac{x+y}{x-y}$$

$$7) \frac{2}{x+3} - \frac{x}{x-1} + \frac{x^2+2}{x^2+2x-3}$$

$$8) \frac{x-6}{x^2-4} - \frac{x-1}{x-2} - \frac{x+1}{x+2}$$

$$9) \frac{x^3+8}{x^2-4} \times \frac{x^2-4x+4}{x^2-2x+4}$$

$$10) \frac{x^2-16}{2x+6} \times \frac{x+3}{x-4}$$

$$11) \frac{x^3+3x}{x^2-9} \div \frac{x^2+5x-14}{x^2+4x-21}$$

$$12) \frac{x^2+5x+6}{x-3} \div \frac{x^2-x-6}{x+3}$$

## \*Variation

Find the variation constant and the variation equation for the following.

- 1)  $y$  varies inversely as  $x$ ,  $y = 1$  when  $x = 8$ .
- 2)  $y$  varies directly as  $x$ ,  $y = 0.4$  when  $x = 10$ .
- 3)  $y$  varies jointly as  $x$  and  $z$ ,  $y = 50$  when  $x = 8$ .
- 4)  $y$  varies jointly as  $x$  and  $z$  and inversely as the product of  $w$  and  $q$ ,

$$x = 4 \text{ when } z = \frac{1}{3}, w = 5, y = 6, \text{ and } q = 6.$$

## Exponents

Simplify

1)  $x^2 y^{-4} z^4^{-3}$

2)  $-2xy^3 \quad 4x^3 y^{-3}$

3)  $\left( \frac{14x^4 y^2 z^6}{-7x^2 y^3 z^6} \right)$

4)  $\left( \frac{4^{-2}}{2^{-6}} \right)^2$

5)  $\left( \frac{2^{-1} x^{-3} y^5 z^{-4}}{3^{-2} x^4 y^6 z^{-3}} \right)^{-2}$

6)  $\frac{3x^5 y^3 z^2^2}{-3x^{-2} y^3 z^{-3}^4}$

7)  $\frac{x^{-2} y^{-3}^{-4}}{x^2 y^3^{-3}}$

## Radical Expressions

Simplify

1)  $\left(x^{\frac{2}{3}}\right)^{\frac{3}{5}}$

2)  $\left(\frac{7^{-\frac{1}{3}}}{7^{-\frac{1}{2}}}\right)$

3)  $\sqrt[3]{27x^6}$

4)  $\sqrt[4]{x^3}\sqrt[3]{x}$

5)  $\sqrt[3]{\frac{64x^6}{27}}$

6)  $\frac{\sqrt[3]{60xy^3}}{\sqrt[3]{10x}}$

7)  $\frac{\sqrt[3]{x^2}}{\sqrt[4]{x}}$

8)  $\sqrt{50} + 2\sqrt{18} + \sqrt{32}$

9)  $\sqrt{5} - 4\sqrt{8} \quad \sqrt{5} + 6\sqrt{8}$

10)  $2 - \sqrt{8}^2$

## Complex Numbers

Simplify

1)  $4 + 3i + 5 - 9i$

2)  $-5 - 6i - -6 + 6i$

3)  $2 + i \quad 4 - i$

4)  $6 + i^2$

5)  $\frac{6 - i}{3}$

6)  $\frac{4 + 5i}{6i}$

7)  $\frac{4 + 6i}{2 - i}$

8)  $i^{40}$

9)  $i^{33}$

10)  $i^2 - i - i^3 + i^4$

## Quadratic/Radical Equations

Solve

1)  $3x^2 - 4x - 1 = 0$

2)  $x^2 + 6x + 9 = 64$

3)  $x - 9^2 = 81$

4)  $x^2 - 10x - 4 = 0$

5)  $x^2 - 6x + 13 = 0$

6)  $\sqrt{5x-4} = 7 - \sqrt{13-x}$

7)  $\sqrt{4x-8} = -8$

8)  $\sqrt{2x-6} = 8$

9)  $\sqrt{4x-4} - 1 = \sqrt{x+4}$

10)  $\frac{1}{x} + \frac{1}{x+2} = \frac{1}{8}$

11)  $x - 3^2 + x - 1^2 = 0$

12)  $\frac{4}{x} - \frac{x}{7} = \frac{8}{3}$

## Formulas

Solve for the following variables.

1)  $V = \frac{1}{3}s^2h; s$

2)  $a^2 + b^2 + c^2 = d^2; b$

3)  $N = \frac{k^2 - 3k}{2}; k$

4)  $A = 2\pi r^2 + 2\pi rh; r$

5)  $I = \sqrt{\frac{1}{GT}}; T$

6)  $\frac{t}{a} + \frac{t}{b} = 1; b$

7)  $I = \frac{nE}{E + nr}; E$

## Graphing Circles

Given the radius and center, graph the following circles.

1)  $r = 2, C(4, 3)$

2)  $r = 4, C(0, 3)$

## Functions

Find the domain of the following functions.

1)  $f(x) = x^2 - 4$

2)  $g(x) = \frac{x-5}{x^2-5x+6}$

3)  $h(x) = x^3$

## Applications

1) The base of a triangular sail is 5 ft less than its height.

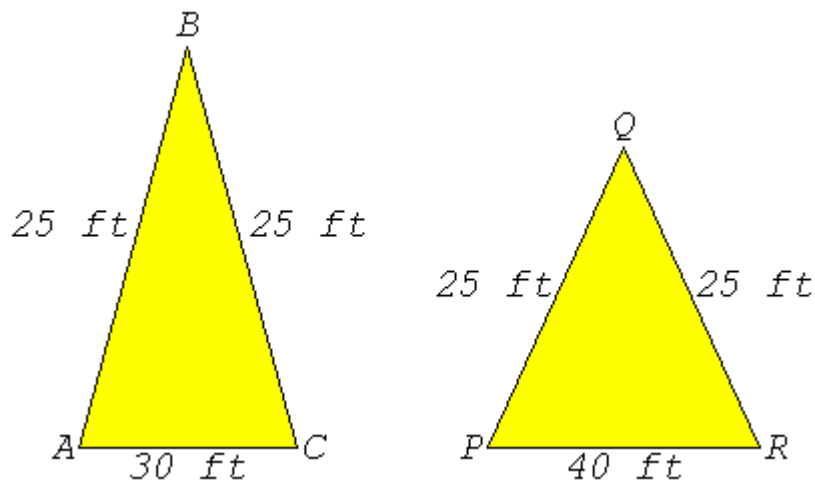
The area is  $42 \text{ ft}^2$ . Find the base and the height of the sail.

2) Jimmy flies his plane 400 mi at a certain speed. Andy flies his plane 1200 mi at a speed that is 100 mph faster, but it takes 2 hours longer.

Find the speed of each plane.

3) Paul takes 3 hours longer to paint the door than Pete does. When they work together, it takes them 2 hours. How long would each take to do the job alone?

4) Which triangle has a greater area and by how much?



5) John has nickels, quarters, and dimes totaling \$3.30, 21 coins in all, with one more nickel than dimes. How many of each type of coin does he have?

6) If for all  $x \neq -1$ ,  $\frac{3x^2 + kx + 2}{x + 1} = 3x + 2$ , then find  $k$ .

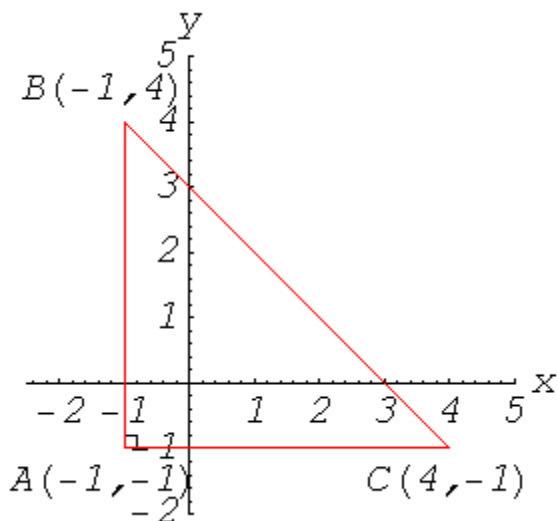
7) Find the sums of the solutions to  $4x^2 + 10x - 6 = 0$ .

8) If  $\frac{6}{\sqrt{x^2 + 4}} = 2$ , then  $x^2 = ?$

9) Given the point  $A(-2, 3)$ . Find the coordinates of point  $B$ , such that the line  $x = 4$  is a perpendicular bisector of  $\overline{AB}$ .

10) Find the area of the following right triangle. Find the distance from  $A$  to  $B$ .

Find the distance from  $B$  to  $C$ . Find the midpoint from  $B$  to  $C$ .



Created by: Ivan Temesvari

## Intermediate Algebra Math Workshop Solutions

### Order of Operations with Real Numbers

- 1)  $53/105$
- 2)  $99/20$
- 3)  $-207$
- 4)  $360$
- 5)  $-296$
- 6)  $-10$
- 7)  $5/42$
- 8)  $-4$
- 9)  $37$
- 10)  $127/2$

### Solve and Graph Linear Equations/Inequalities in One Variable

- 1)  $x = -\frac{11}{2}$
- 2)  $x = -25$
- 3)  $x = 10$
- 4)  $x = -14$
- 5)  $x = \frac{93}{73}$
- 6)  $x = -1$
- 7)  $x = \frac{29}{21}$
- 8)  $x < 14$
- 9)  $x < 5$
- 10)  $x > -\frac{2}{3}$
- 11)  $x \geq 0$
- 12)  $x > -9$
- 13)  $x \leq -6$

### Compound Inequalities

- 1)  $-\infty, \infty$
- 2)  $-10, \infty$
- 3)  $-\infty, 20$
- 4)  $3, 7$
- 5)  $\emptyset$
- 6)  $0 < x < 7$
- 7)  $-1 \geq x > -\frac{11}{3}$

### Absolute Value Equations/Inequalities

- 1)  $\emptyset$
- 2)  $5, 0$
- 3)  $\left\{\frac{9}{5}, 1\right\}$
- 4)  $5, 1$
- 5)  $3, -3$
- 6)  $-4, 4$
- 7)  $-\infty, -2 \cup 10, \infty$
- 8)  $\left[\frac{2}{3}, \frac{10}{3}\right]$
- 9)  $\left(-\infty, -\frac{7}{2}\right] \cup \left[-\frac{5}{2}, \infty\right)$

### Distance Formula

- 1)  $\sqrt{34}$
- 2)  $\sqrt{101}$

### Midpoint Formula

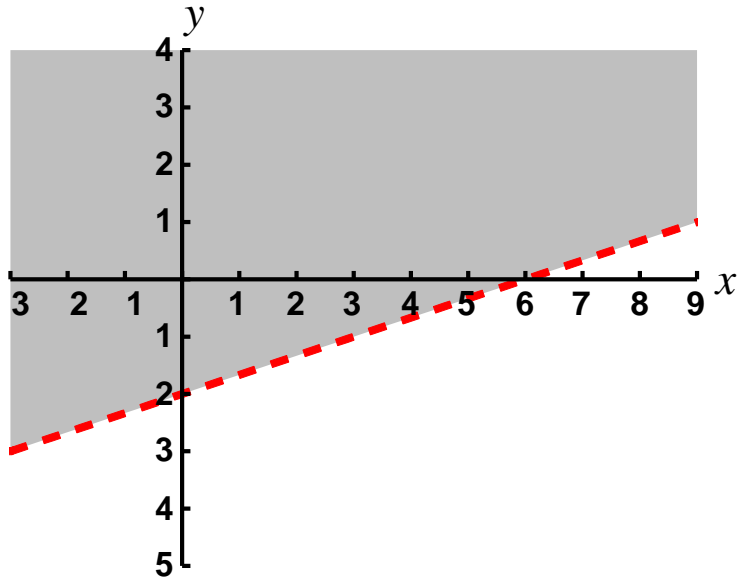
- 1)  $\left(0, -\frac{1}{2}\right)$
- 2)  $\left(\frac{1}{2}, -\frac{7}{2}\right)$

### Concepts of Lines

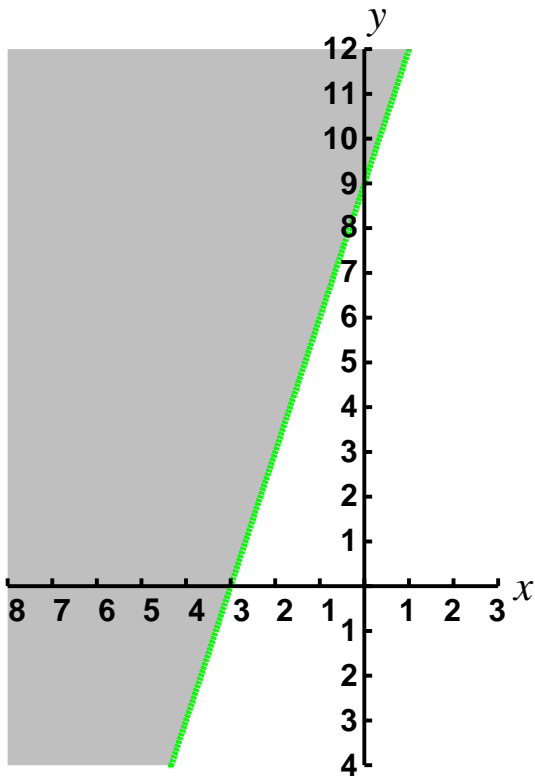
- 1)  $y = -\frac{2}{3}x + 2$
- 2)  $y = 3x - 12$
- 3)  $y = 9x + 5$
- 4)  $y = -2x - 3$
- 5)  $\parallel: y = \frac{2}{5}x + \frac{13}{5}, \perp: y = -\frac{5}{2}x + \frac{11}{2}$
- 6)  $\parallel: y = \frac{2}{3}x + \frac{14}{3}, \perp: y = -\frac{3}{2}x + \frac{5}{2}$

Solve and Graph Linear Inequalities in Two Variables

1)



2)



Solve and Graph Systems of Linear Equations/Inequalities in Two Variables

1)  $5, 0$

2)  $\left(\frac{25}{7}, -\frac{30}{49}\right)$

3)  $-21, 21$

4)  $2, 3$

5)  $\left(\frac{1}{5}, -\frac{7}{5}\right)$

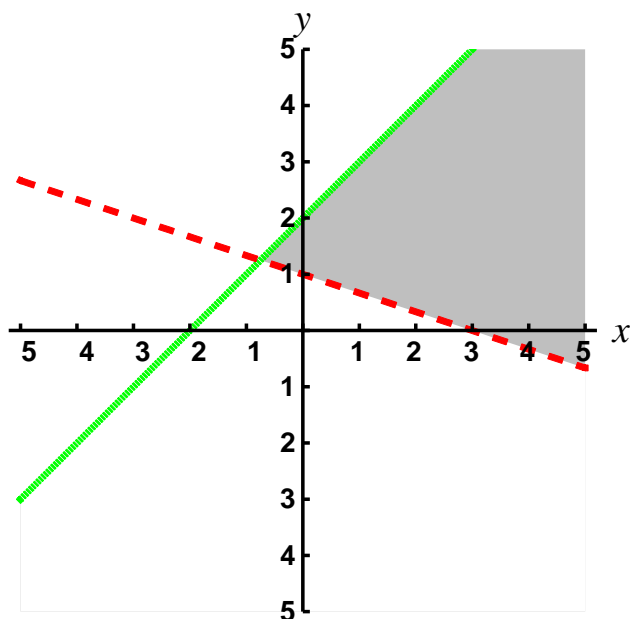
6)  $3, -2, 1$

7)  $\left(\frac{5}{4}, \frac{27}{2}, -18\right)$

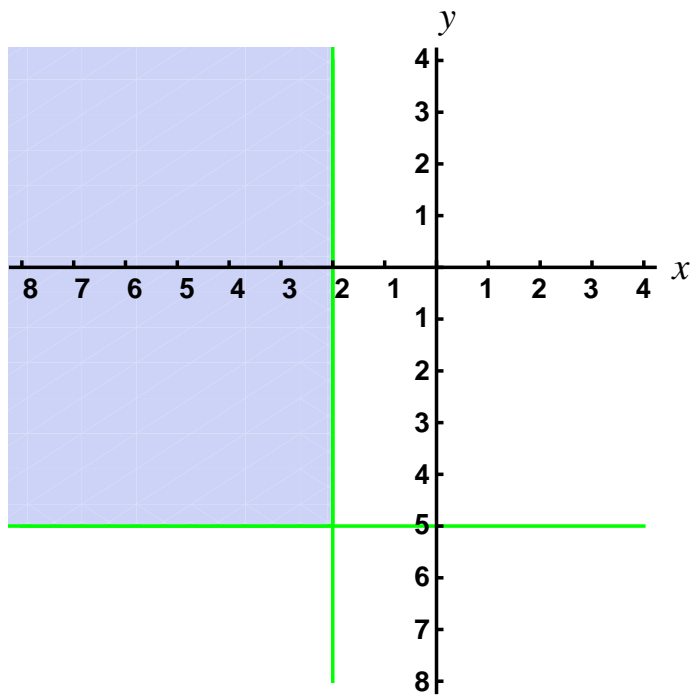
8)  $\left(8, \frac{18}{5}, \frac{42}{5}\right)$

9)  $\left(\frac{55}{7}, -\frac{26}{7}, -\frac{15}{7}\right)$

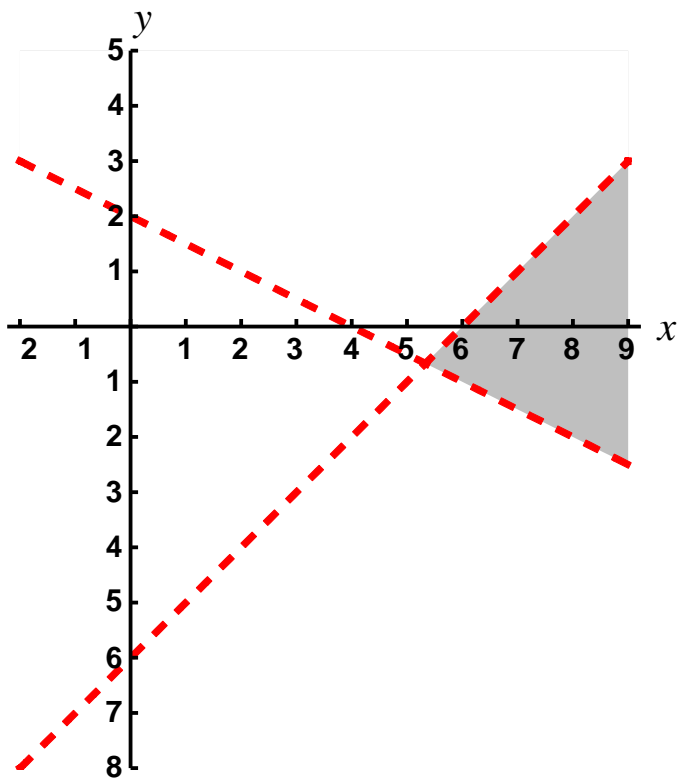
10)



11)



12)



### Basic Operations with Polynomials

1)  $-13x^2y^2 - 12x^2y - xy^2 + 6xy$

2)  $x^2 + 5$

3)  $9x^2 - 30x + 25$

4)  $x^3 - 9x^2 + 27x - 27$

5)  $4x^2 - 25$

6)  $18x^2 + 45x - 50$

7)  $x^3 - 1$

8)  $x^2 - \frac{2}{15}x - \frac{8}{15}$

9)  $y^2 - 5y + 25$

10)  $x + 5 - \frac{45}{x + 5}$

11)  $x^2 + 2x + \frac{12}{x - 2}$

12)  $2x^3 + 7x - 5 + \frac{29x - 6}{x^2 - 4}$

### Factor Polynomials

1)  $(x + 2)(x + 3)$

2)  $(2x - 5)(x - 6)$

3)  $(4xy - 1)(4xy + 1)$

4)  $(y - 1)(y + 1)(y^2 + 1)$

5)  $(5x - 3)^2$

6)  $(6x - 1)(2x - 5)$

7)  $(7x + 5)(3x - 7)$

8)  $5(x - 1)(x - 7)$

9)  $4x(x + 1)^2$

10)  $(x + 1)(x^2 - x + 1)$

11)  $(x - 1)(x^2 + x + 1)$

12)  $8(2xy - 1)(4x^2y^2 + 2xy + 1)$

13)  $(3x^2 + y^2)(9x^4 - 3x^2y^2 + y^4)$

### Basic Operations with Rational Expressions

1) 1

2)  $\frac{2x}{x-1}$

3)  $\frac{-2(3x+1)}{x-1}$

4)  $\frac{2(x^2+11)}{(x+4)(x-5)}$

5)  $\frac{2x+21}{x^2-4}$

6)  $\frac{x^2+4xy+y^2}{x^2-y^2}$

7)  $\frac{-x}{(x+3)(x-1)}$

8)  $\frac{-2x^2+x-2}{(x-2)(x+2)}$

9)  $x-2$

10)  $\frac{x+4}{2}$

11)  $\frac{x(x^2+3)}{(x-2)(x+3)}$

12)  $\frac{(x+3)^2}{(x-3)^2}$

### \*Variation

1)  $y = \frac{8}{x}$

2)  $y = 0.04x$

3)  $y = \frac{25}{16}xz$

4)  $y = \frac{135xz}{wq}$

### Exponents

$$1) \frac{y^{12}}{x^6 z^{12}}$$

$$2) -8x^4$$

$$3) \frac{-2x^2}{y}$$

$$4) 16$$

$$5) \frac{4x^{14} z^2 y^2}{81}$$

$$6) \frac{x^{18} z^{16}}{9y^6}$$

$$7) x^{14} y^{21}$$

### Radical Expressions

$$1) \frac{1}{\sqrt[5]{x^2}}$$

$$2) \sqrt[6]{7}$$

$$3) 3x^2$$

$$4) x^{12} \sqrt{x}$$

$$5) \frac{4x^2}{3}$$

$$6) y^3 \sqrt[3]{6}$$

$$7) \sqrt[12]{x^5}$$

$$8) 15\sqrt{2}$$

$$9) -187 + 4\sqrt{10}$$

$$10) 12 - 8\sqrt{2}$$

### Complex Numbers

- 1)  $9 - 6i$
- 2)  $1 - 12i$
- 3)  $9 + 2i$
- 4)  $35 + 12i$
- 5)  $2 - \frac{1}{3}i$
- 6)  $\frac{5}{6} - \frac{2}{3}i$
- 7)  $\frac{2}{5} + \frac{16}{5}i$
- 8)  $1$
- 9)  $i$
- 10)  $0$

### Quadratic/Radical Equations

- 1)  $\left\{ \frac{1}{3} 2 - \sqrt{7}, \frac{1}{3} 2 + \sqrt{7} \right\}$
- 2)  $-11,5$
- 3)  $18,0$
- 4)  $5 \pm \sqrt{29}$
- 5)  $3 \pm 2i$
- 6)  $4$
- 7)  $\emptyset$
- 8)  $5$
- 9)  $7 \pm \sqrt{65}$
- 10)  $2 \pm i$
- 11)  $\left\{ \frac{2}{3} - 14 \pm \sqrt{259} \right\}$

### Formulas

$$1) s = \sqrt{\frac{3V}{h}}$$

$$2) b = \sqrt{d^2 - a^2 - c^2}$$

$$3) k = \frac{1}{2} (3 \pm \sqrt{9 + 8N})$$

$$4) r = \frac{-h\sqrt{\pi} + \sqrt{2A + \pi h^2}}{2\sqrt{\pi}}$$

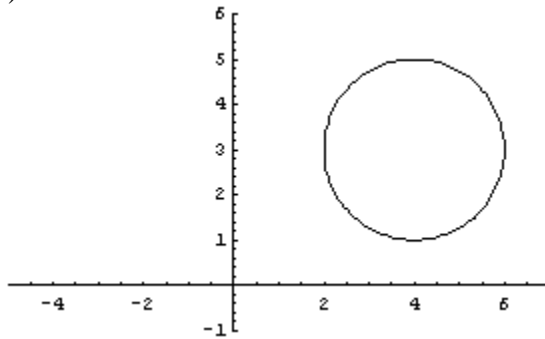
$$5) T = \frac{1}{I^2 G}$$

$$6) b = \frac{-at}{t-a}$$

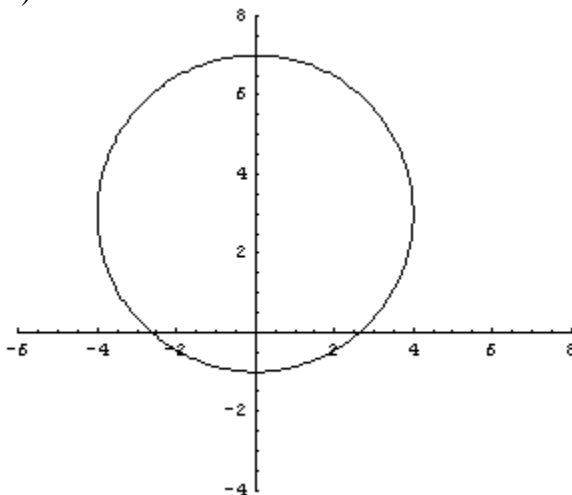
$$7) E = \frac{Inr}{n-I}$$

### Graphing Circles

1)



2)



Functions

- 1)  $\mathbb{R}$
- 2)  $\{x \mid x^2 \geq 2 \text{ and } x^2 \leq 3\}$
- 3)  $\mathbb{R}$

Applications

- 1)  $b = 7 \text{ ft}, h = 12 \text{ ft}$
- 2)  $r_1 = 200 \text{ mph}, t_1 = 2 \text{ hrs}, r_2 = 300 \text{ mph}, t_2 = 4 \text{ hrs}$
- 3) Paul takes 6 hours and Pete takes 3 hours.
- 4) They both have the same area.
- 5) 6 nickels, 5 dimes, 10 quarters
- 6)  $k = 5$
- 7)  $-\frac{5}{2}$
- 8) 5
- 9)  $(10, 3)$
- 10)  $A = 12.5 \text{ units squared}, d(A, B) = 5, d(B, C) = 5\sqrt{2}, \text{ midpoint} = \left(\frac{35}{2}, \frac{3\sqrt{2}}{2}\right)$