



Math Workshop Trigonometry

Workshop Objective: Once a student successfully completes the Prealgebra Math Workshop, s/he will achieve a basic understanding the material taught in a MAT 122 course offered at Oakton. Students will be offered the skills and knowledge required of them to demonstrate competency on the Math Placement Test in any of the topics covered during the workshop.

Workshop Topics: Trigonometric Functions, Inverse Trigonometric Functions, Trigonometric Identities, Trigonometric Equations, Trigonometric Form of Complex Numbers, Roots and Powers of Complex Numbers, Solving Right Triangles, Law of Sines, Law of Cosines, Polar Equations and Graphs, Parametric Equations and Graphs, Vectors, Dot Products, Applications.

Workshop Overview: The workshop will identify and explain all of the various topics listed above. The topics covered during the workshop are found on the Oakton Math Placement Test. The supplementary exercises provided are a review of what a student would expect to learn in a MAT 122 course. **Any exercise sets labeled with an asterisk (*) are additional topics not necessarily found on the Math Placement Test.**

Once a student feels confident with all of these workshop topics, then another, more advanced Math Workshop is recommended. A student may re-take any workshop. Each student has an opportunity to re-take the Math Placement Test one time, if they are not satisfied with their test performance. Please feel free to discuss any of your concerns with your Math Workshop instructor. This workshop is not intended to be a substitute for taking/completing MAT 122 over a normal semester setting. Command in the topic material, and supplementary exercises, is not a guarantee that a student has qualified to place into a higher math course (above MAT 122). However, if a student does have a solid understanding of the topics covered, then that student should place into a higher math course (above MAT 122).

Trigonometric Functions

Graph over $[-2\pi, 2\pi]$. Identify all of the intercepts and asymptotes.

1) $t_1(x) = \sin(x)$

2) $t_2(x) = \cos(x)$

3) $t_3(x) = \tan(x)$

4) $t_4(x) = \cot(x)$

5) $t_5(x) = \sec(x)$

6) $t_6(x) = \csc(x)$

7) $t_7(x) = \sin\left(x + \frac{\pi}{2}\right)$

8) $t_8(x) = \cos(x - \pi)$

9) $t_9(x) = 3\sin(x) + 1$

10) $t_{10}(x) = -2\cos(x) - 2$

11) $t_{11}(x) = 2\tan(x - \pi)$

12) $t_{12}(x) = 3 + \frac{1}{2}\tan x$

13) $t_{13}(x) = -2 - \cot x$

14) $t_{14}(x) = \csc\left(x + \frac{\pi}{3}\right)$

15) $t_{15}(x) = 2 + 3\sec(x - \pi)$

*Inverse Trigonometric Functions

Graph.

1) $t_1(\theta) = \cos^{-1} \theta$

2) $t_2(\theta) = \sin^{-1} \theta$

3) $t_3(\theta) = \tan^{-1} \theta$

4) $t_4(\theta) = \sec^{-1} \theta$

5) $t_5(\theta) = \csc^{-1} \theta$

6) $t_6(\theta) = \cot^{-1} \theta$

Trigonometric Identities

$$a) \sin^2 x + \cos^2 x = 1$$

$$b) 1 + \cot^2 x = \csc^2 x$$

$$c) \tan^2 x + 1 = \sec^2 x$$

$$d) \cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$e) \cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$f) \sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$g) \sin(A - B) = \sin A \cos B - \sin B \cos A$$

$$h) \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$i) \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Verify the following identities.

$$1) \frac{\csc \theta + \cot \theta}{\tan \theta + \sin \theta} = \cot \theta \csc \theta$$

$$2) \frac{1 - \sin \theta}{1 + \sin \theta} = \sec^2 \theta - 2 \sec \theta \tan \theta + \tan^2 \theta$$

$$3) \frac{\sin^2 \theta}{\cos \theta} = \sec \theta - \cos \theta$$

$$4) \sec \alpha - \tan \alpha^2 = \frac{1 - \sin \alpha}{1 + \sin \alpha}$$

$$5) \frac{\cos \theta + 1}{\tan^2 \theta} = \frac{\cos \theta}{\sec \theta - 1}$$

$$6) \tan^2 \alpha \sin^2 \alpha = \tan^2 \alpha + \cos^2 \alpha - 1$$

Trigonometric/Inverse Trigonometric Equations

Solve on $[0, 2\pi)$.

1) $7 \csc \theta - 2 = 5$

2) $\cos^2 x + 2 \cos x + 1 = 0$

3) $2 \sin^2 x = \sin x$

4) $\sin 2x + \sin x = 0$

5) $\cos 2x = \sqrt{2} - \cos 2x$

6) $\cos \theta - \sin \theta = 0$

7) $\sin \theta = -\sqrt{2} - \cos \theta$

8) $3 \cot^2 \theta - 4 \csc \theta = 1$

9) $\cos 2x + \sin x = 1$

*10) $\frac{4}{3} \cos^{-1} \frac{y}{4} = \pi$

*11) $\arccos x + 2 \arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}$

*12) $\sin^{-1} x + \tan^{-1} \sqrt{3} = \frac{2\pi}{3}$

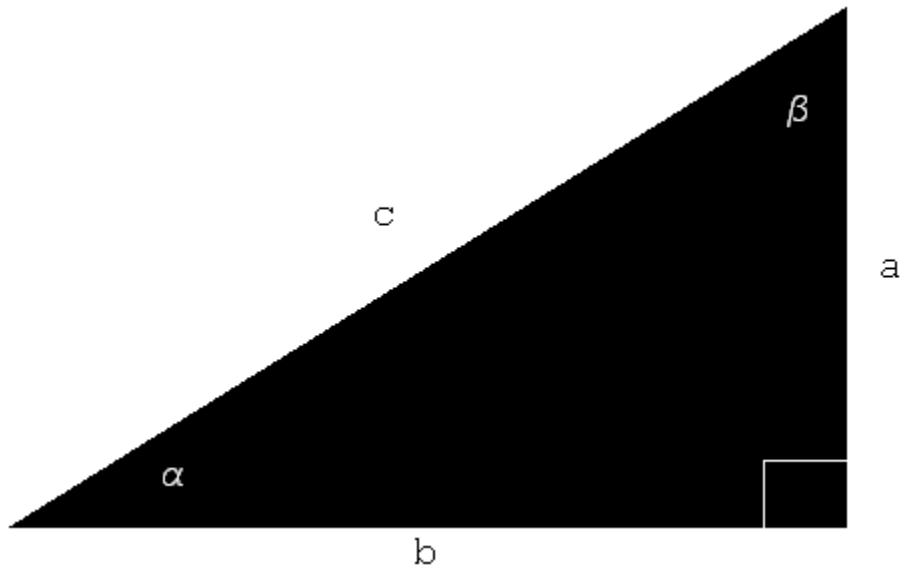
13) $4 \sin \theta \cos \theta = \sqrt{3}$

14) $\tan 3x + \sec 3x = 2$

15) $\cos 2x - \cos x = 0$

Solving Right Triangles

Solve the following right triangles.

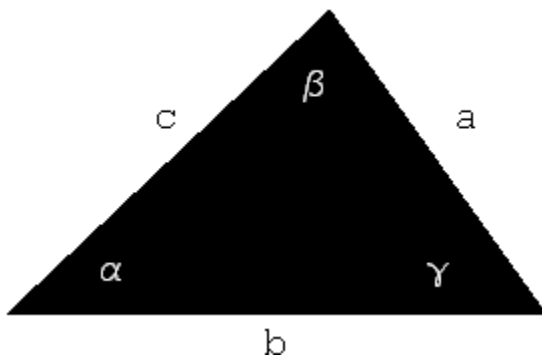


1) $\alpha = 25^\circ$ and $a = 10$

2) $\beta = 10^\circ$ and $b = 6$

Law of Sines

Solve the following triangles.



1) $a = 11$, $\gamma = 35^\circ$, and $\beta = 16^\circ$

2) $b = 24.23$, $\gamma = 110.4^\circ$, and $\beta = 37.9^\circ$

3) $c = 8$, $\alpha = 28^\circ$, and $\beta = 52^\circ$

4) $a = 6.36$, $b = 3.59$, and $\beta = 16.4^\circ$

Law of Cosines

Solve the following triangles.

1) $a = 6.5$, $b = 8.2$, $c = 10.5$

2) $a = 6$, $b = 8$, $\gamma = 116^\circ$

Find the area of each triangle.

3) $b = 9$ feet, $c = 6$ feet, $\alpha = 83^\circ$

4) $a = 12$ inches, $b = 15$ inches, $\gamma = 52^\circ$

*Polar Equations and Graphs

Plot the following points.

1) $\left(-2, \frac{\pi}{3}\right)$

2) $\left(3, -\frac{\pi}{4}\right)$

Convert the following polar coordinates to rectangular coordinates.

3) $5, 30^\circ$

4) $\left(-2, \frac{\pi}{4}\right)$

Convert the following rectangular coordinates to polar coordinates.

5) $-1, -1$

6) $\sqrt{3}, 1$

Graph the following polar equations.

7) $r = 2 \sin \theta$

8) $r = 3$

9) $r \cos \theta = 1$

10) $r = 3 + \cos \theta$

11) $r = \frac{2}{1 - \cos \theta}$

*Trigonometric Form of Complex Numbers

Graph the following and express in trigonometric/polar form.

- 1) 2
- 2) $3i$
- 3) $-15 - 20i$
- 4) $3 - 4i$

Find the following.

- 5) $5 \cos 20^\circ + i \sin 50^\circ \cdot 2 \cos 10^\circ + i \sin 250^\circ$
- 6) $7 \cos 45^\circ + i \sin 30^\circ \cdot 7 \cos 45^\circ + i \sin 60^\circ$

Convert to rectangular form.

- 7) $5 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$
- 8) $2 \cos 330^\circ + i \sin 330^\circ$

*Roots and Powers of Complex Numbers

Find.

- 1) $\sqrt{3} + i$ ⁵
- 2) $1 + i$ ²⁰
- 3) $-\sqrt{3} + i$ ⁶
- 4) $\left[\sqrt{2} \left(\cos \left(\frac{3\pi}{4} \right) + i \sin \left(\frac{3\pi}{4} \right) \right) \right]^4$

Solve.

- 5) $x^4 + 16 = 0$
- 6) $x^5 - i = 0$

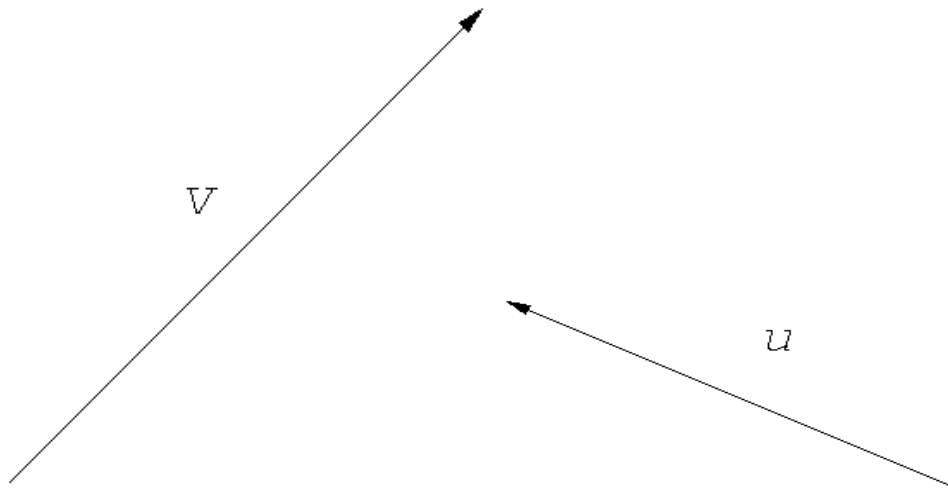
*Parametric Equations and Graphs

Graph. Find the rectangular equation for each.

- 1) $x = t, y = t + 2, -4 \leq t \leq 6$
- 2) $x = 4t - 1, y = t^2 - 1, -2 \leq t \leq 4$
- 3) $x = 3 \sin t, y = 3 \cos t, 0 \leq t \leq 2\pi$

*Vectors

Use the following vectors \vec{v} and \vec{u} to draw the following.



- 1) $u + v$
- 2) $u - v$
- 3) $3u$
- 4) $-2v$

Find the position vector \mathbf{v} with initial point P and terminal point Q.
Write in the form of $ai + bj$.

- 5) P 2,4 , Q -4,5
- 6) P -1,4 , Q 0,5

Find the following.

Given: $v = 2i + 3j, u = -4i + 5j$

- 7) $u + v$
- 8) $3u - 4v$

Find the magnitude $\|\mathbf{v}\|$ of the following vectors.

- 9) $v = 3i + 4j$
- 10) $v = -2i + 4j$

Write the vector in the form $ai + bj$ given its magnitude and the angle it makes with the positive x-axis.

11) $\|v\| = 4, \alpha = 60^\circ$

12) $\|v\| = 3, \alpha = 180^\circ$

***Dot Products**

Find the dot products of the following.

Find the angle θ between the two vectors. Use the formula $u \cdot v = \|u\| \cdot \|v\| \cos \theta$.

1) $u = -8i + 9j$ and $v = -2i - 4j$

2) $u = 3i + 6j$ and $v = -5i + 6j$

Determine whether the following vectors are parallel, orthogonal, or neither.

3) $v = 2i + j, w = i - 2j$

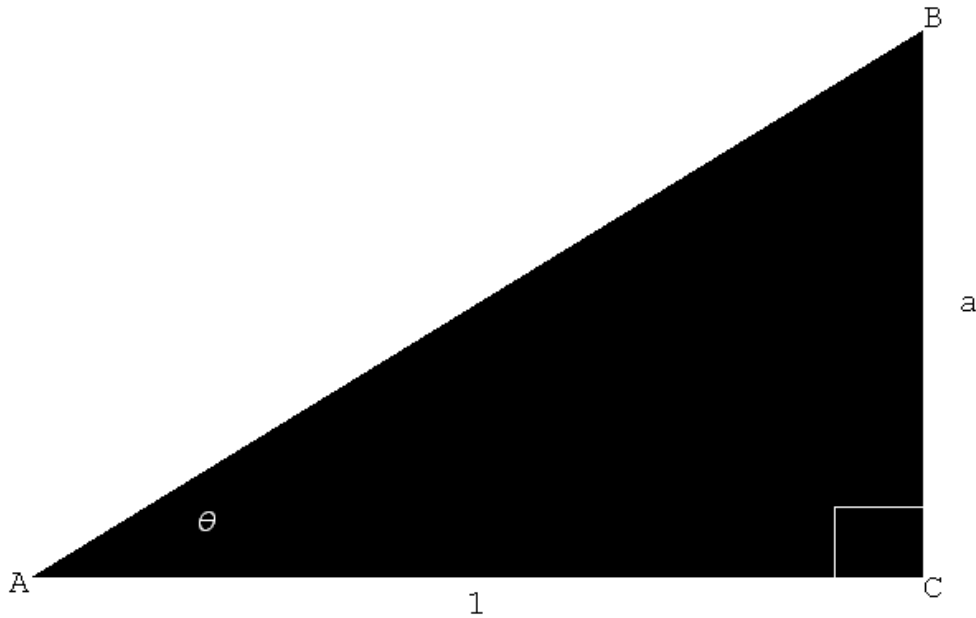
4) $v = 2i - j, w = 4i - 2j$

Applications

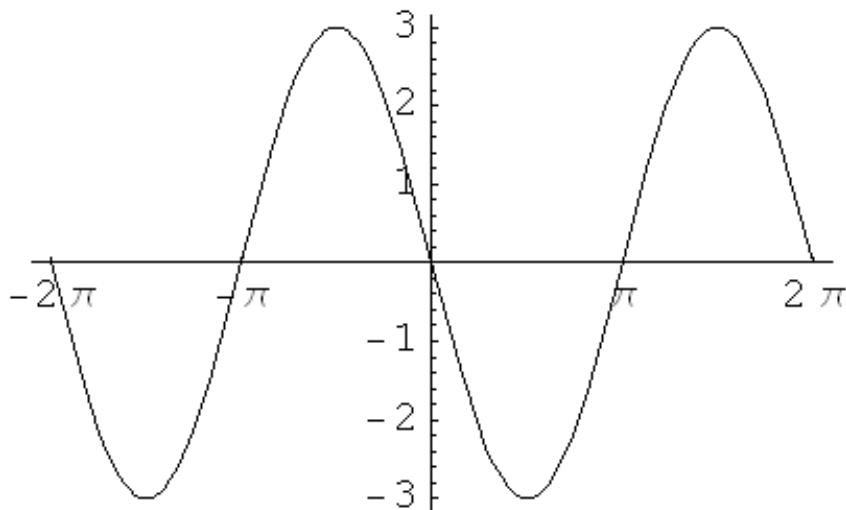
- 1) Find the force required to pull a 25-pound weight up a ramp inclined at 42° to the horizontal.
- 2) Find the work done to push a 9-pound box up a 35° incline 5 feet.
- 3) A plane has an airspeed of 520 mph. The pilot wishes to fly on a bearing of 310° . A wind of 37 mph is blowing from the East. What direction should the pilot fly and what will be her actual speed?
- 4) A pipeline is to run between points A and B, which are separated by a protected wetlands area. To avoid the wetlands, the pipe will run from point A to C and then to B. The distances involved are $AB=150$ km, $AC=102$ km, and $BC=135$ km. What angle should be used at point C?
- 5) If $\cos\theta = \frac{1}{3}$, where θ is the measure of an acute angle, what is the exact value for $\sin\theta$?
- 6) Find all real values of x for which the expression is defined, $\frac{\cos x}{\sin x} = ?$
- 7) If $\sin 2x = \frac{1}{2}$, and $0^\circ \leq x < 90^\circ$, what is $\sin 2x$? Round to the nearest ten-thousandths place.
- 8) If $\sin\theta = \frac{1}{2}$ and $\cos\theta = \frac{\sqrt{3}}{2}$, then $\cos 2\theta = ?$ Find the exact value.
- 9) What is the exact value of $\cos\left(\frac{\pi}{8}\right)$?

Use the half-angle formula: $\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos\alpha}{2}}$.
- 10) If $\frac{3\pi}{2} \leq \theta < 2\pi$ and $\sin\theta = \frac{-\sqrt{3}}{2}$, then $\theta = ?$

- 11) In the right triangle below $\triangle ABC$, the length of side \overline{AC} is 1 and the angle θ is such that $30^\circ \leq \theta \leq 60^\circ$. What are the possible values for a , and the length of side \overline{BC} ?



- 12) A 12-foot ladder leans against a house making an angle of 50° with the ground. How many feet up the house does the ladder reach?
- 13) If θ is the measure of an interior angle of $\triangle ABC$ and $\sin \theta = \cos \theta$, then $\theta = ?$
- 14) Find the exact value of the following: $\sin 30^\circ + \cos 60^\circ + \tan 45^\circ$.
- 15) Find an equation for the trigonometric function graphed in the standard x, y coordinate plane below.

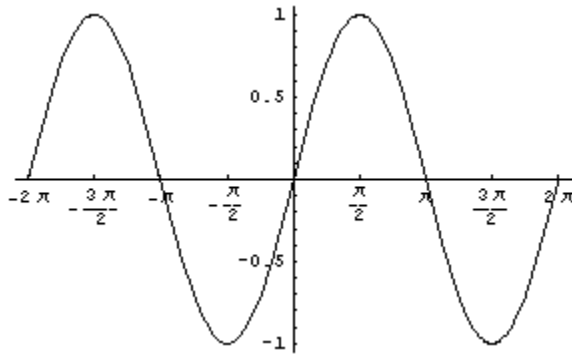


Created by: Ivan Temesvari

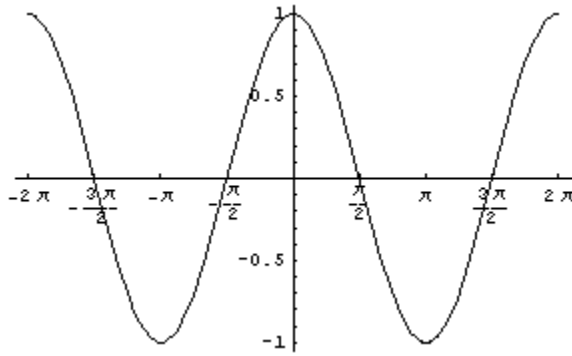
Trigonometry Math Workshop Solutions

Trigonometric Functions

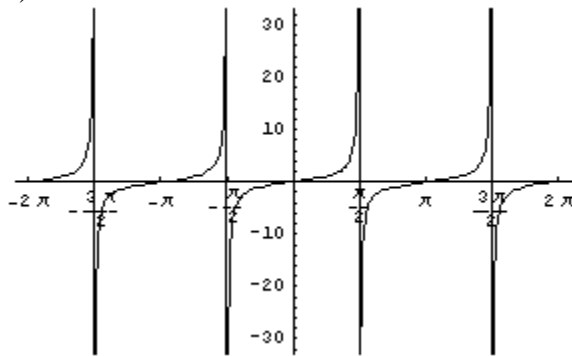
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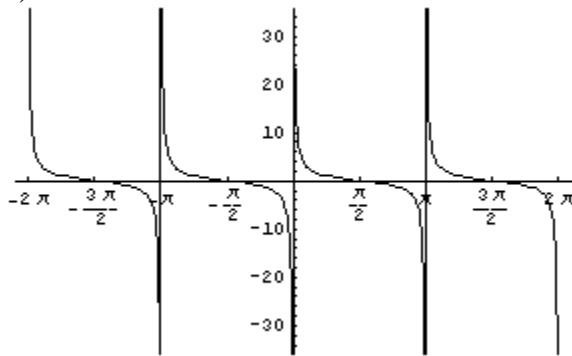
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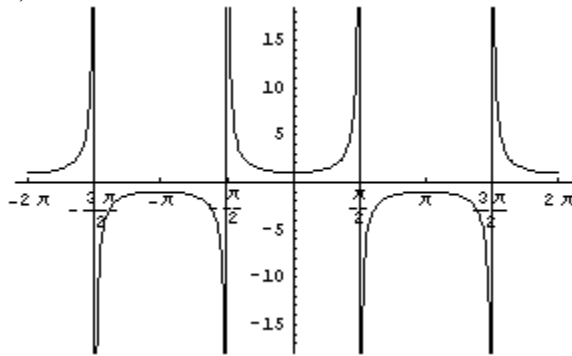
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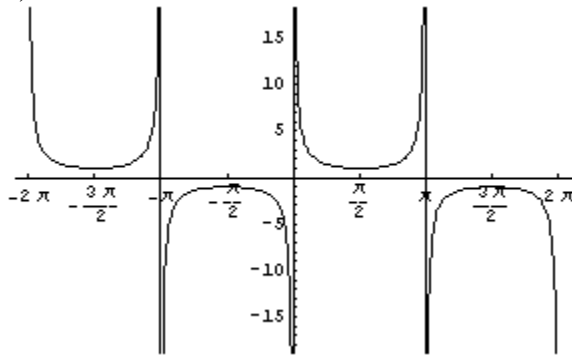
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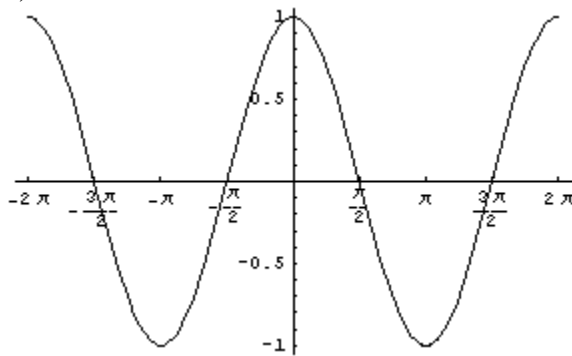
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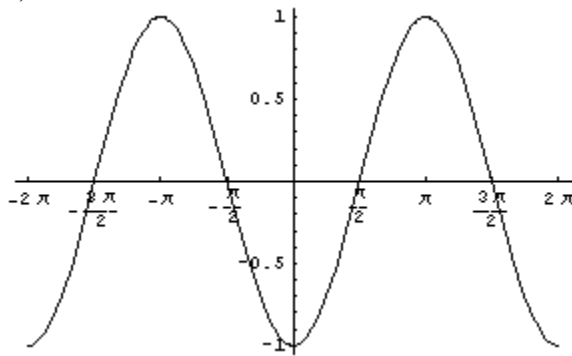
6)



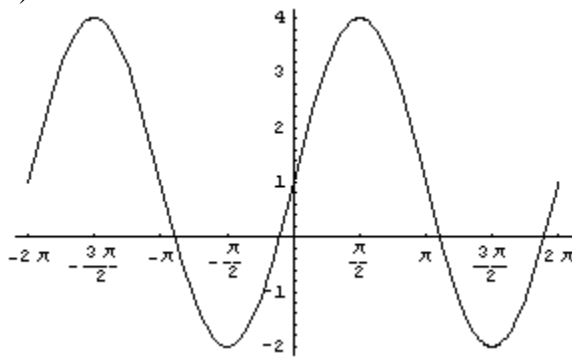
7)



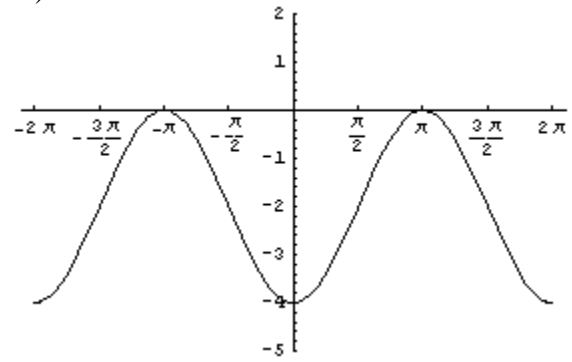
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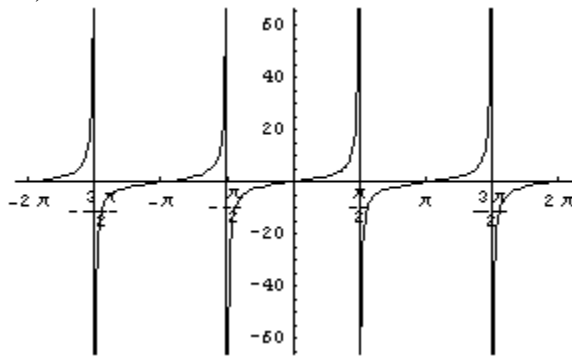
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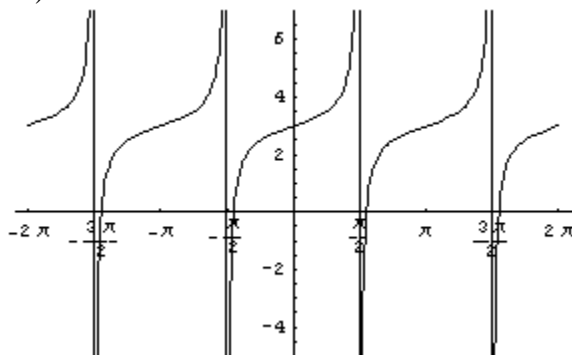
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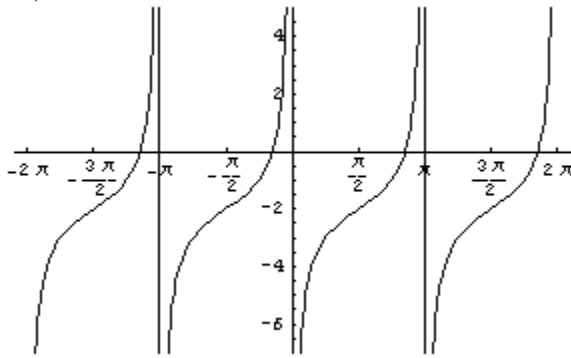
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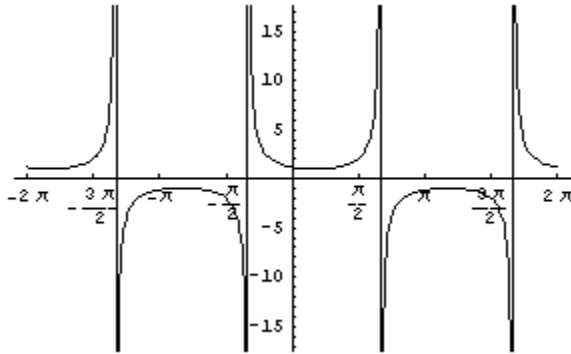
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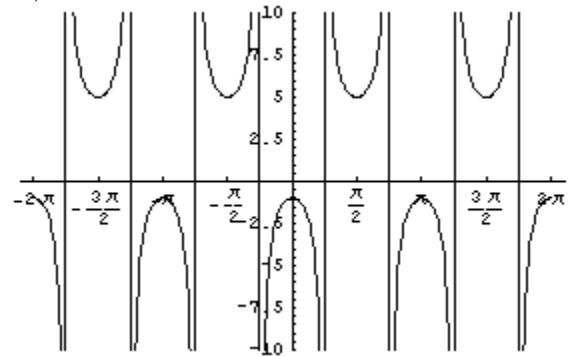
13)



14)

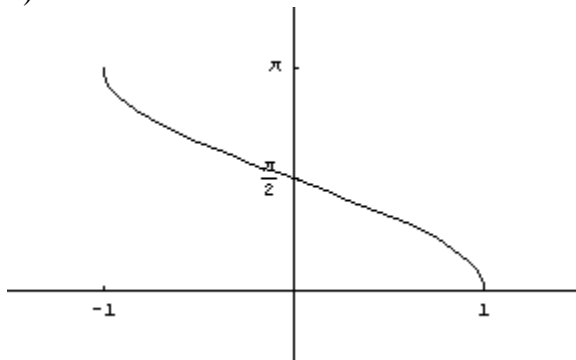


15)

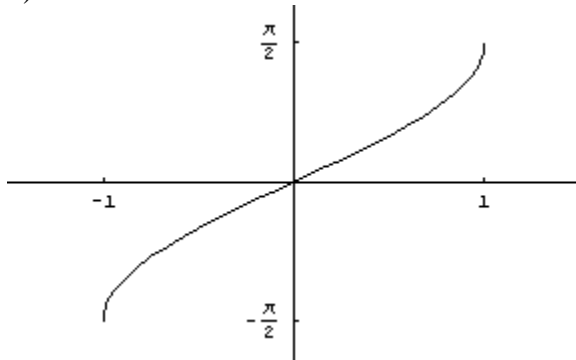


Inverse Trigonometric Functions

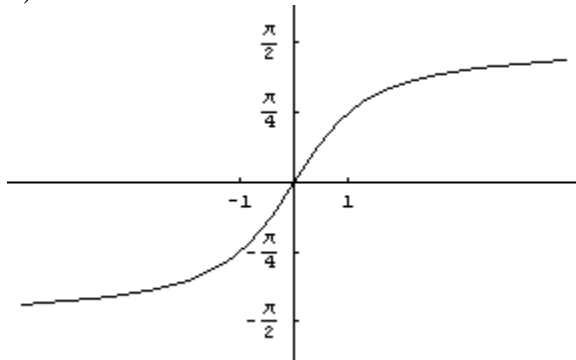
1)



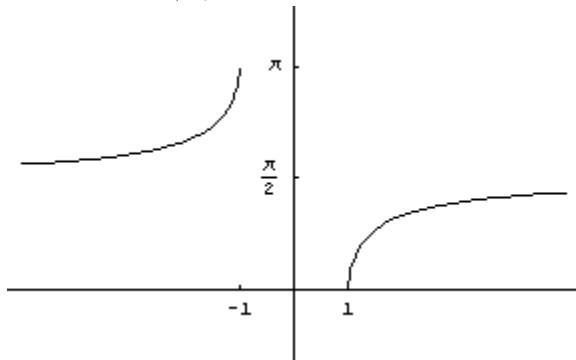
2)



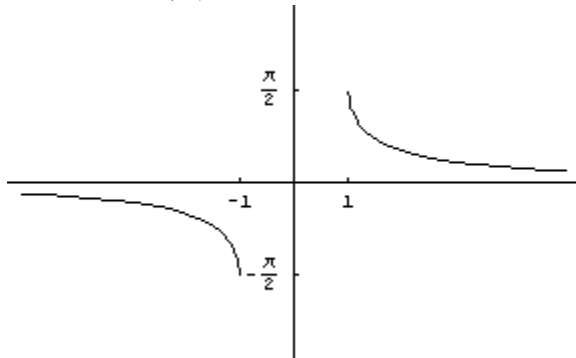
3)



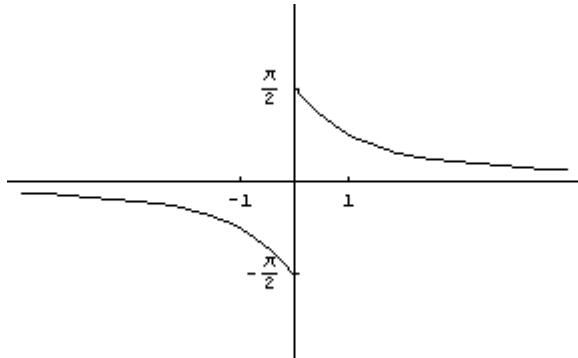
4) $y = \cos^{-1}\left(\frac{1}{x}\right)$ on TI-83



5) $y = \sin^{-1}\left(\frac{1}{x}\right)$ on TI-83



$$6) y = \left(\tan^{-1}\left(\frac{1}{x}\right) \right) * (x \leq 0) + \left(\tan^{-1}\left(\frac{1}{x}\right) + \pi \right) * (x > 0) \text{ on TI-83}$$



Trigonometric Identities

$$1) \Rightarrow \frac{\csc \theta + \cot \theta}{\tan \theta + \sin \theta} = \frac{\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \sin \theta} = \frac{\frac{1 + \cos \theta}{\sin \theta}}{\frac{\sin \theta + \sin \theta \cos \theta}{\cos \theta}} = \frac{1 + \cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta + \sin \theta \cos \theta}$$

$$= \frac{\cos \theta}{\sin \theta} \cdot \frac{1 + \cancel{\cos \theta}}{\sin \theta (\cancel{1 + \cos \theta})} = \cot \theta \csc \theta$$

$$2) \Leftarrow \frac{1}{\cos^2 \theta} - \frac{2}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1 - 2 \sin \theta - \sin^2 \theta}{\cos^2 \theta} = \frac{(-\sin \theta)^2}{1 - \sin^2 \theta}$$

$$= \frac{(-\cancel{\sin \theta})(-\cancel{\sin \theta})}{(-\cancel{\sin \theta})(\cancel{+ \sin \theta})} = \frac{1 - \sin \theta}{1 + \sin \theta}$$

$$3) \Leftarrow \frac{1}{\cos \theta} - \cos \theta = \frac{1 - \cos^2 \theta}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta}$$

$$4) \Rightarrow (\sec \alpha - \tan \alpha)^2 = \sec^2 \alpha - 2 \sec \alpha \tan \alpha + \tan^2 \alpha \Rightarrow \text{follow \#2}$$

$$5) \Leftarrow \frac{\cos \theta}{\sec \theta - 1} \cdot \frac{\sec \theta + 1}{\sec \theta + 1} = \frac{\cos \theta \cdot \left(\frac{1}{\cos \theta} + 1 \right)}{\sec^2 \theta - 1} = \frac{1 + \cos \theta}{\tan^2 \theta}$$

$$6) \Rightarrow \frac{\sin^2 \alpha}{\cos^2 \alpha} \cdot \sin^2 \alpha = \frac{1 - \cos^2 \alpha}{\cos^2 \alpha} \cdot \sin^2 \alpha = \left(\frac{1}{\cos^2 \alpha} - 1 \right) \cdot \sin^2 \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha} - \sin^2 \alpha$$

$$= \tan^2 \alpha - (-\cos^2 \alpha) \tan^2 \alpha + \cos^2 \alpha - 1$$

Trigonometric Equations

- 1) $\frac{\pi}{2}$
- 2) $\pm \pi$
- 3) $\left\{0, \frac{\pi}{6}, \pi, \frac{5\pi}{6}\right\}$
- 4) $\left\{0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$
- 5) $\left\{\frac{\pi}{8}, \frac{15\pi}{8}\right\}$
- 6) $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$
- 7) $\frac{5\pi}{4}$
- 8) $\frac{\pi}{6}$
- 9) $\left\{0, -\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \pi\right\}$
- 10) $-2\sqrt{2}$
- 11) \emptyset
- 12) $\frac{\sqrt{3}}{2}$
- 13) $30^\circ, 60^\circ, 210^\circ, 240^\circ$
- 14) $.2145, 2.3089, 4.4033$
- 15) $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

Trigonometric Identities

Solving Right Triangles

- 1) $\alpha = 25^\circ, \beta = 65^\circ, c = 23.66$
- 2) $a = 34.03, \alpha = 80^\circ, c = 34.55$

Law of Sines

- 1) $\alpha = 129^\circ, b = 3.90, c = 8.12$
- 2) $\alpha = 148.3^\circ, a = 20.73, c = 36.97$
- 3) $\gamma = 100^\circ, b = 6.40, a = 3.81$
- 4) $\gamma = 133.59^\circ, \alpha = 30.01^\circ, c = 9.21$

Law of Cosines

1) $\gamma = 90.41^\circ$, $\beta = 51.35^\circ$, $\alpha = 38.24^\circ$

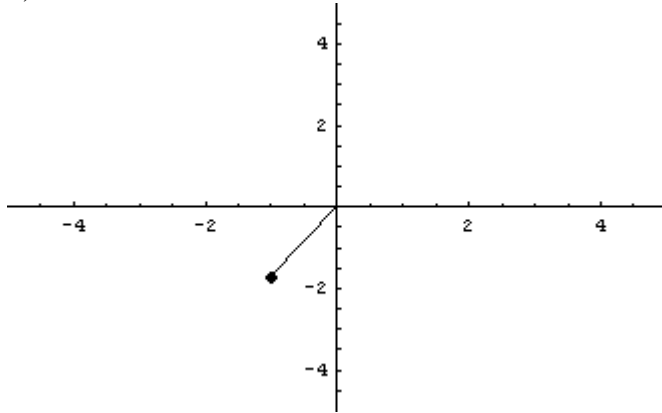
2) $c = 11.92$, $\beta = 37.10^\circ$, $\alpha = 130.98^\circ$

3) $A = 26.80 \text{ ft}^2$

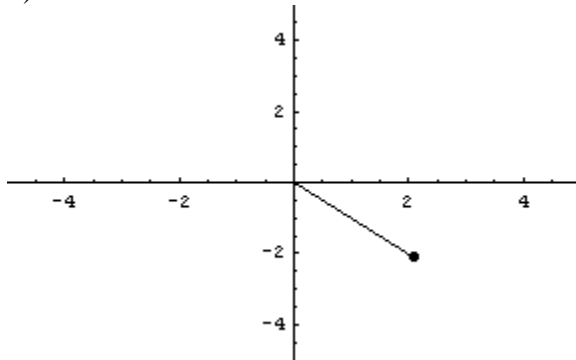
4) $A = 70.92 \text{ in}^2$

Polar Equations and Graphs

1)



2)



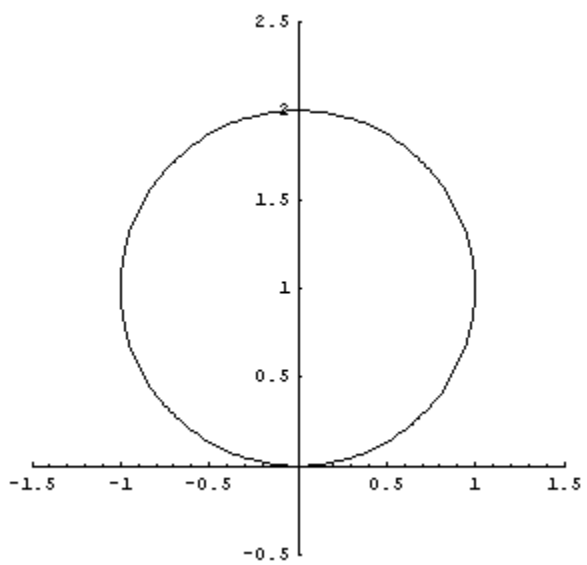
3) $x = 4.33$, $y = 2.5$

4) $x = -1.41$, $y = -1.41$

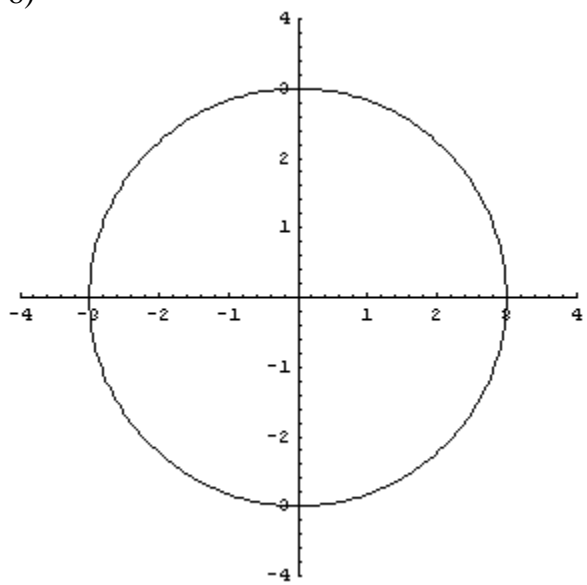
5) $\sqrt{2}, 45^\circ$

6) $2, 30^\circ$

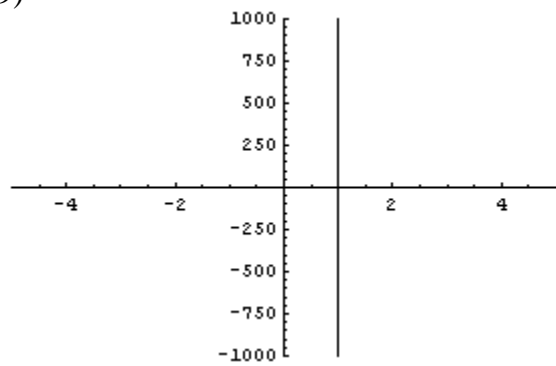
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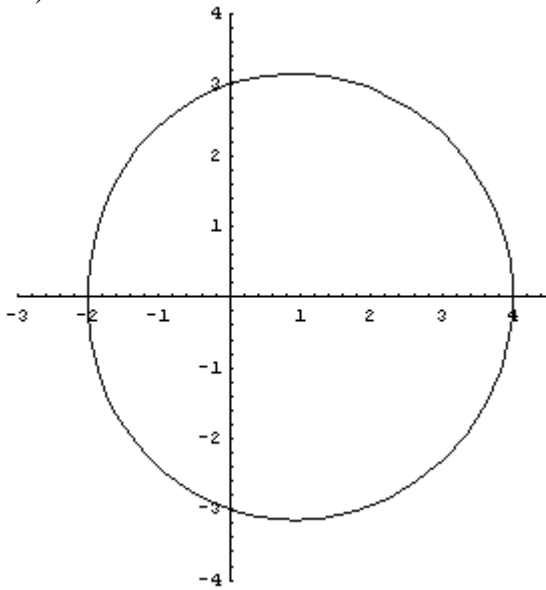
8)



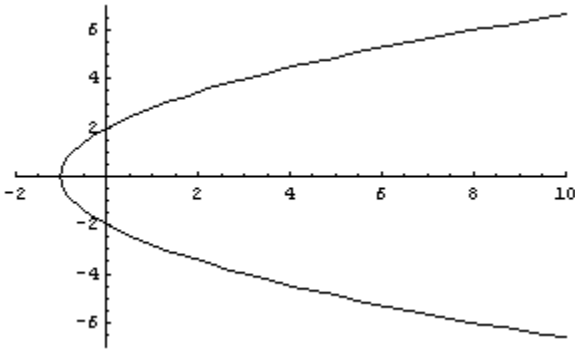
9)



10)



11)



Trigonometric Form of Complex Numbers

1) $2, 0^\circ$

2) $\left(3, \frac{\pi}{2}\right)$

3) $-2.5, 53.13^\circ$

4) $5, -53.13^\circ$

5) $-10i$

6) $49\text{cis}105^\circ \approx -12.68 + 47.33i$

7) $\frac{5}{2} + \frac{5\sqrt{3}}{2}i$

8) $\sqrt{3} - i$

Roots and Powers of Complex Numbers

1) $16i - 16\sqrt{3}$

2) -1024

3) -64

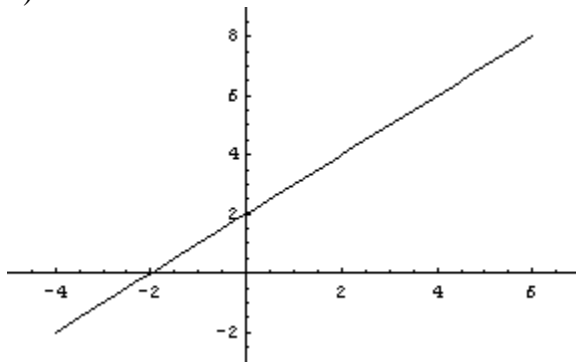
4) -4

5) $2 \cos 45^\circ + i \sin 45^\circ$, $2 \cos 135^\circ + i \sin 135^\circ$, $2 \cos 225^\circ + i \sin 225^\circ$, $2 \cos 315^\circ + i \sin 315^\circ$

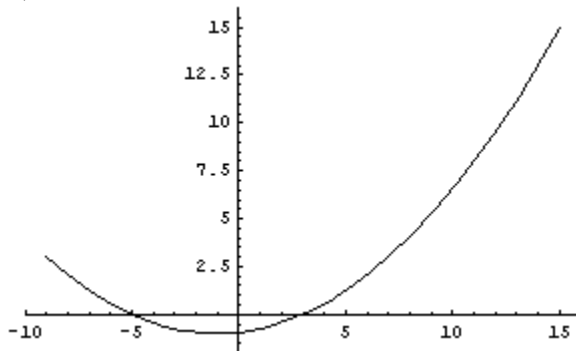
6) $\cos 18^\circ + i \sin 18^\circ$, $\cos 90^\circ + i \sin 90^\circ$, $\cos 162^\circ + i \sin 162^\circ$, $\cos 234^\circ + i \sin 234^\circ$, $\cos 306^\circ + i \sin 306^\circ$

Parametric Equations and Graphs

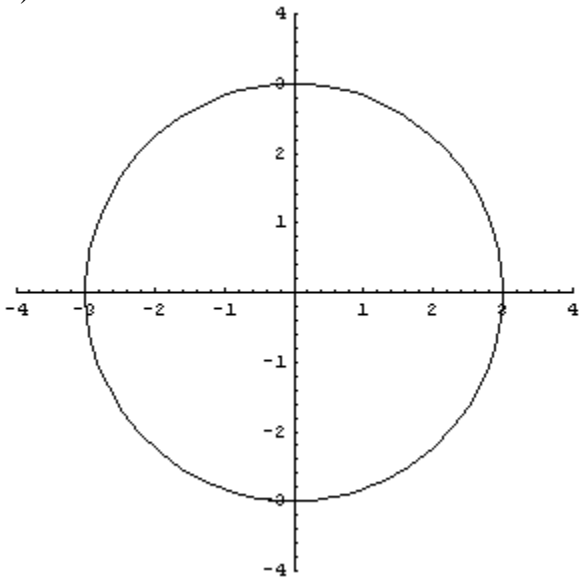
1)



2)

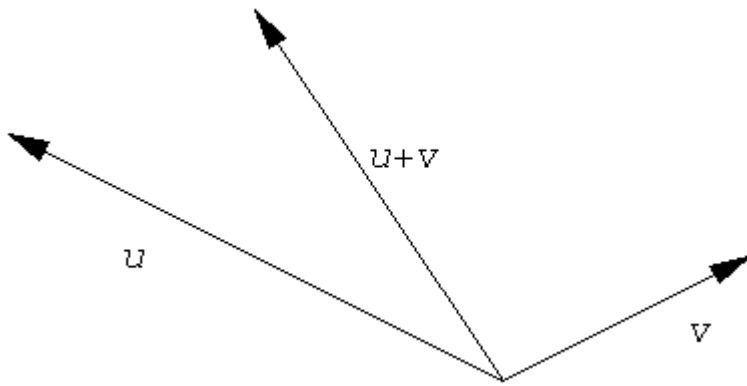


3)

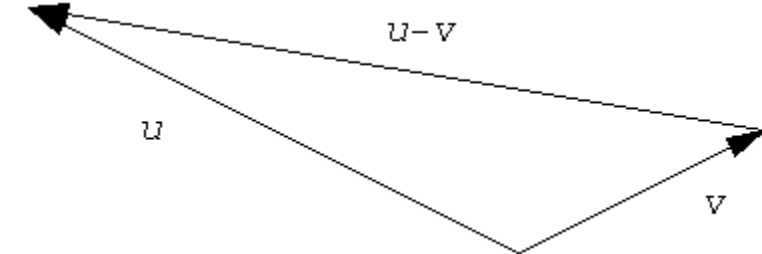


Vectors

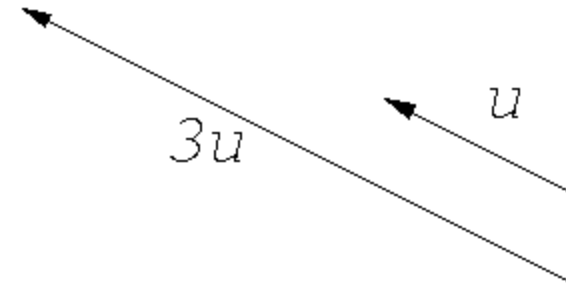
1)



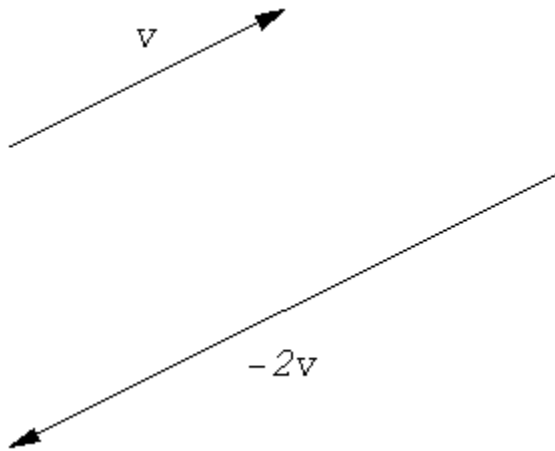
2)



3)



4)



5) $\langle -6, 1 \rangle$

6) $\langle 1, 1 \rangle$

7) $\langle -2, 8 \rangle$

8) $\langle -20, 3 \rangle$

9) 5

10) $2\sqrt{5}$

11) $2i + 2\sqrt{3}j$

12) $-3i$

Dot Products

1) $\theta = 111.80^\circ$

2) $\theta = 57.47^\circ$

3) orthogonal

4) not orthogonal

Applications

- 1) $F = 16.73 \text{ lb}$
- 2) $W = 25.8 \text{ ft} \cdot \text{lb}$
- 3) Bearing is 307.38° at 547.78 mph
- 4) $\theta = 77.14^\circ$
- 5) $\frac{2\sqrt{2}}{3}$
- 6) $x|x \neq k\pi$
- 7) $\frac{1}{2}\sqrt{2+\sqrt{3}} \approx .9659$
- 8) $\frac{1}{2}$
- 9) $\frac{1}{2}\sqrt{2+\sqrt{2}}$
- 10) 300°
- 11) $\frac{1}{\sqrt{3}} \leq a \leq \sqrt{3}, \frac{1}{\cos 30^\circ} \leq \overline{BA} \leq 2$
- 12) $h = 9.19 \text{ ft}$
- 13) 45°
- 14) 2
- 15) $y = -\sin x$

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