Parallel Circuits

Topics Covered in Chapter 5

5-1: The Applied Voltage $V_A$ Is the Same Across Parallel Branches

5-2: Each Branch $I$ Equals $V_A / R$

5-3: Kirchhoff’s Current Law (KCL)

5-4: Resistance in Parallel

5-5: Conductances in Parallel
Topics Covered in Chapter 5

- 5-6: Total Power in Parallel Circuits
- 5-7: Analyzing Parallel Circuits with Random Unknowns
- 5-8: Troubleshooting: Opens and Shorts in Parallel Circuits
Characteristics of a Parallel Circuit

- Voltage is the same across each branch in a parallel circuit.
- The total current is equal to the sum of the individual branch currents.
- The equivalent resistance \( R_{EQ} \) is less than the smallest branch resistance. The term equivalent resistance refers to a single resistance that would draw the same amount of current as all of the parallel connected branches.
- Total power is equal to the sum of the power dissipated by each branch resistance.
A parallel circuit is formed when two or more components are connected across the same two points.

A common application of parallel circuits is the typical house wiring of many receptacles to the 120-V 60 Hz ac power line.
5-1: The Applied Voltage $V_A$ Is the Same Across Parallel Branches

Fig. 5-1: Example of a parallel circuit with two resistors. (a) Wiring diagram. (b) Schematic diagram.

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The current in a parallel circuit equals the voltage applied across the circuit divided by the resistance between the two points where the voltage is applied.

Each path for current in a parallel circuit is called a branch. Each branch current equals \( V/R \) where \( V \) is the same across all branches.
Fig. 5-3: Parallel circuit. (a) the current in each parallel branch equals the applied voltage $V_A$ divided by each branch resistance $R$. 

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5-3: Kirchhoff’s Current Law (KCL)

Components connected in parallel are usually wired across one another, with the entire parallel combination connected to the voltage source.

Fig. 5-5a: The current in the main line equals the sum of the branch currents. Note that from G to A at the top of this diagram is the negative side of the main line, and from B to F at the bottom is the positive side. (a) Wiring diagram. Arrows inside the lines indicate current in the main line for \( R_1 \); arrows outside indicate current for \( R_2 \).
This circuit structure gives the same result as wiring each parallel branch directly to the voltage source.

The main advantage of using this structure is that it requires less wire.
The pair of leads connecting all the branches to the voltage source terminals is the main line.

All the current in the circuit must come from one side of the voltage source and return to the opposite side for a complete path.

The amount of current in the main line is equal to the total of the branch currents.
The total current $I_T$ in the main line is equal to the sum of the branch currents.

This is known as Kirchhoff’s current law (KCL).

It applies to any number of parallel branches, whether the resistances in those branches are equal or not.
5-3: Kirchhoff’s Current Law (KCL)

\[ I_T = I_1 + I_2 + I_3 + I_4 \]
The combined equivalent resistance of a parallel circuit may be found by dividing the common voltage across all resistances by the total current of all the branches.

\[ R_{EQ} = \frac{V_A}{I_T} \]
A combination of parallel branches is called a bank.

A combination of parallel resistances $R_{EQ}$ for the bank is always less than the smallest individual branch resistance because $I_T$ must be more than any one branch current.
The equivalent resistance of a parallel circuit must be less than the smallest branch resistance.

Adding more branches to a parallel circuit reduces the equivalent resistance because more current is drawn from the same voltage source.
Fig. 5-7: How adding parallel branches of resistors increases \( I_T \) but decreases \( R_{EQ} \). (a) One resistor. (b) Two branches. (c) Three branches. (d) Equivalent circuit of the three branches in (c).

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5-4: Resistance in Parallel

- Total Current and Reciprocal Resistance Formulas
  - In a parallel circuit, the total current equals the sum of the individual branch currents:

\[ I_T = I_1 + I_2 + I_3 + \ldots + \text{etc.} \]

- Total current is also equal to total voltage divided by equivalent resistance:

\[ I_T = \frac{V_T}{R_{EQ}} \]
5-4: Resistance in Parallel

- Total Current and Reciprocal Resistance Formulas
  - The equivalent resistance of a parallel circuit equals the reciprocal of the sum of the reciprocals:

\[
R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \text{etc.}}
\]

Equivalent resistance also equals the applied voltage divided by the total current:

\[
R_{EQ} = \frac{V_A}{I_T}
\]
5-4: Resistance in Parallel

- Determining the Equivalent Resistance

\[ R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \]

\[ R_{EQ} = 4 \, \Omega \]

Fig. 5-8: Two methods of combining parallel resistances to find \( R_{EQ} \). (a) Using the reciprocal resistance formula to calculate \( R_{EQ} \) as 4 \( \Omega \). (b) Using the total line current method with an assumed line voltage of 20 V gives the same 4 \( \Omega \) for \( R_{EQ} \).
### 5-4: Resistance in Parallel

- **Special Case: Equal Value Resistors**
  - If $R$ is equal in all branches, divide one resistor’s value by the number of resistors.

  \[
  R_{EQ} = \frac{R}{N}
  \]

  \[
  R_{EQ} = \frac{60 \text{ k}\Omega}{3 \text{ resistors}}
  \]

  \[
  R_{EQ} = 20 \text{ k}\Omega
  \]

  ![Image of resistance in parallel](image)

  Fig. 5-9: For the special case of all branches having the same resistance, just divide $R$ by the number of branches to find $R_{EQ}$. Here, $R_{EQ} = 60 \text{ k}\Omega / 3 = 20 \text{ k}\Omega$. 

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5-4: Resistance in Parallel

- Special Case: Two Unequal Resistors
  - When there are only two branches in a parallel circuit and their resistances are unequal, use the formula:

  \[ R_{EQ} = \frac{R_1 \times R_2}{R_1 + R_2} \]

  ![Parallel Circuit Diagram]

  Fig. 5-10: For the special case of only two branch resistances, of any values, \( R_{EQ} \) equals their product divided by the sum. Here, \( R_{EQ} = \frac{2400}{100} = 24\,\Omega \).

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To find an unknown branch resistance, rewrite the formula as follows to solve for the unknown value.

\[ R_X = \frac{R \times R_{EQ}}{R - R_{EQ}} \]

These formulas may be used to simplify complex circuits.
Conductance \((G)\) is equal to \(1 / R\).

Total (equivalent) conductance of a parallel circuit is given by:

\[ G_T = G_1 + G_2 + G_3 + \ldots + \text{etc.} \]
5-5: Conductances in Parallel

- Determining Conductance
  - Each value of $G$ is the reciprocal of $R$. Each branch current is directly proportional to its conductance.
  - Note that the unit for $G$ is the siemens (S).
5-5: Conductances in Parallel

\[ G_1 = \frac{1}{20 \, \Omega} = 0.05 \, \text{S} \]

\[ G_2 = \frac{1}{5 \, \Omega} = 0.2 \, \text{S} \]

\[ G_3 = \frac{1}{2 \, \Omega} = 0.5 \, \text{S} \]

\[ G_T = 0.05 + 0.2 + 0.5 = 0.75 \, \text{S} \]
Total power is equal to the sum of the power dissipated by the individual resistances of the parallel branches:

\[ P_T = P_1 + P_2 + P_3 + \ldots + \text{etc.} \]

Total power is equal to voltage times total current:

\[ P_T = V_T I_T \]
### 5-6: Total Power in Parallel Circuits

#### Determining Power

\[ P_1 = \frac{10^2}{10 \, \Omega} = 10 \, \text{W} \]

\[ P_2 = \frac{10^2}{5 \, \Omega} = 20 \, \text{W} \]

\[ P_T = 10 + 20 = 30 \, \text{W} \]

Check: \( P_T = V_T \times I_T = 10 \, \text{V} \times 3 \, \text{A} = 30 \, \text{W} \)

Fig. 5-14: The sum of the power values \( P_1 \) and \( P_2 \) used in each branch equals the total power \( P_T \) produced by the source.
When the voltage across one branch is known, use this voltage for all branches. There can be only one voltage across branch points with the same potential difference.

If the values for $I_T$ and one branch current ($I_1$) are known, the value of $I_2$ can be found by subtracting $I_1$ from $I_T$. 
Opens in Parallel Circuits

- An open circuit in one branch results in no current through that branch.
- However, an open circuit in one branch has no effect on the other branches. This is because the other branches are still connected to the voltage source.
- An open in the main line prevents current from reaching any branch, so all branches are affected.
5-8: Troubleshooting: Opens and Shorts in Parallel Circuits

- Opens in Parallel Circuits.

- In part b bulbs 2 and 3 still light. However, the total current is smaller. In part a no bulbs light.

Fig. 5-16: Effect of an open in a parallel circuit. (a) Open path in the main line—no current and no light for all the bulbs. (b) Open path in any branch—bulb for that branch does not light, but the other two bulbs operate normally.

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5-8: Troubleshooting: Opens and Shorts in Parallel Circuits

- Shorts in a Parallel Circuit
  - A short circuit has zero resistance, resulting in excessive current in the shorted branch.
  - A shorted branch shorts the entire circuit.
  - Current does not flow in the branches that are not shorted. They are bypassed by the short circuit path that has zero resistance.
**5-8: Troubleshooting:**

**Opens and Shorts in Parallel Circuits**

- **A Short in a Parallel Circuit**

![Diagram of a short circuit in a parallel circuit]

- The other branches are shorted out. The total current is very high.

Fig. 5-17: Effect of a short circuit across parallel branches. (a) Normal circuit. (b) Short circuit across points H and G shorts out all the branches.

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