All problems are worth 10 points for a total of 100. Use of technology is encouraged, but the maximal amount of partial credit can be earned by placing all of your work on the exam.

1.) Display and solve the equations necessary to establish the domain of the following function.

\[ h(x) = \frac{x - 5}{7 - \sqrt{x} + 6} \]

**Solution:** The denominator requires \( 7 - \sqrt{x} + 6 \neq 0 \) which implies \( x \neq 43 \). The radical requires \( x + 6 \geq 0 \) which implies \( x \geq -6 \).

2.) The U.S. population was 180,000,000 in 1960 and 250,000,000 in 1990. Assuming an exponential population growth model, \( f(t) = N_0e^{kt} \), what will be the population in 2025?

**Solution:** If \( t = 0 \) represents 1960, then \( f(0) = 180 \) million which implies \( N_0 = 180 \). The fact that \( f(30) = 250 \) million in 1990 implies \( \frac{25}{18} = e^{30k} \) or \( k = \frac{\ln(25/18)}{30} \approx 0.0109501 \). So, in 2025, \( f(65) = 180e^{65\cdot\ln(25/18)/30} \approx 366.763 \) million.

3.) A ball is thrown straight up into the air with an initial velocity of 32 ft/sec from over the edge of a 384 foot cliff. Its height at time \( t \) is given by \( s(t) = -16t^2 + 32t + 384 \).

a) Display and solve the equation necessary to determine at what time the ball strikes the ground.

**Solution:** \( s(t) = 0 = -16(t^2 - 2t - 24) = -16(t - 6)(t + 4) \) implies \( t = 6 \)

b) Find the average velocity of the ball over the time interval \([t, t + h]\) completely simplifying your answer.

**Solution:**

\[
\frac{s(t+h) - s(t)}{h} = \frac{-16(t+h)^2 + 32(t+h) + 384 - (-16t^2 + 32t + 384)}{h} = \frac{-16t^2 - 32th + 16h^2 + 32h + 384 + 16t^2 - 32t - 384}{h} = \frac{h(-32t + 16h + 32)}{h}
\]

\[ = -32t + 16h + 32 \]

4.) Determine the equation of the line through the point \((7, -4)\) that is perpendicular to the line \(3x - 5y = 6\).

**Solution:** \( 3x - 5y = 6 \) implies that \( y = \frac{3}{5}x - \frac{6}{5} \) so the perpendicular slope is \( m = -\frac{5}{3} \). Then, the point slope equation implies \( y - (-4) = -\frac{5}{3}(x - 7) \) or \( y = -\frac{5}{3}x + \frac{29}{3} \)

5.) If \( f(x) = 7x - 4 \) and \( g(x) = 3x^2 - 5x - 2 \), then what is \( f(g(3)) \) ?

**Solution:** \( f(g(3)) = f(3 \cdot 3^2 - 5 \cdot 3 - 2) = f(10) = 7 \cdot 10 - 4 = 66 \).
6.) The following function is one-to-one for all real values of \( x \neq 7 \). Find the inverse function \( f^{-1}(x) \).

**Solution:** \( y = \frac{9 - 4x}{3x^2 - 5} \) implies \( 35xy - 5y = 9 - 4x \) so \(-9 - 5y = -35xy - 4x \) or \( \frac{-9 - 5y}{-35y - 4} = x \). Therefore \( f^{-1}(x) = \frac{9 + 5x}{3x + 4} \).

7.) Forty years ago, oil production in Texas was given by the table below. Use your calculator to find a logarithmic regression equation \( y = a + b \ln x \) for this data. Let \( x = 60 \) represent 1960, \( x = 70 \) represent 1970, and so forth. You do not need to show any work other than your final answer.

<table>
<thead>
<tr>
<th>Year</th>
<th>Metric Tons (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>40.56</td>
</tr>
<tr>
<td>1970</td>
<td>62.1</td>
</tr>
<tr>
<td>1980</td>
<td>90.1</td>
</tr>
</tbody>
</table>

**Solution:**\( -662.650601 + 171.374043 \ln x \)

8.) a) Find the following limit. One-half credit will be awarded for the correct answer, one-half credit will be awarded for the correct reason. For instance, to read the limit off of your graphing calculator would only be worth 1/2 credit. Algebraic reasons for your answer would be worth the remaining 1/2 credit.

\[ \lim_{x \to 0} 9\theta \cot(\theta) \]

**Solution:**
\[\lim_{\theta \to 0} 9\theta \cot \theta = \lim_{\theta \to 0} 9\theta \cdot \frac{\cos \theta}{\sin \theta} = 9 \cdot \lim_{\theta \to 0} \frac{\theta}{\sin \theta} \cdot \lim_{\theta \to 0} \cos \theta = 9 \cdot 1 \cdot 1 = 9\]

b) Find the following limit. One-half credit will be awarded for the correct answer, one-half credit will be awarded for the correct reason. For instance, to read the limit off of your graphing calculator would only be worth 1/2 credit. Algebraic reasons for your answer would be worth the remaining 1/2 credit.

\[ \lim_{x \to 0} \frac{\sqrt{x + 2} - \sqrt{7}}{x} \]

**Solution:**
\[ \lim_{x \to 0} \frac{\sqrt{x + 2} - \sqrt{7}}{x} = \lim_{x \to 0} \frac{\sqrt{x + 2} - \sqrt{7}}{x} \cdot \frac{\sqrt{x + 2} + \sqrt{7}}{\sqrt{x + 2} + \sqrt{7}} = \lim_{x \to 0} \frac{x}{x(\sqrt{x + 2} + \sqrt{7})} = \frac{1}{2\sqrt{7}} \]

9.) Find the following limit. One-half credit will be awarded for the correct answer, one-half credit will be awarded for the correct reason. For instance, to read the limit off of your graphing calculator would only be worth 1/2 credit. Algebraic reasons for your answer would be worth the remaining 1/2 credit.

\[ \lim_{x \to -\infty} \frac{2x^3 - 4}{7x^3 + x^2 + 8} \]

**Solution:**
\[ \lim_{x \to -\infty} \frac{2x^3 - 4}{7x^3 + x^2 + 8} = \lim_{x \to -\infty} \frac{2x^3}{7x^3} = \lim_{x \to -\infty} \frac{2}{7} = \frac{2}{7} \]

10.) Find the following limit. One-half credit will be awarded for the correct answer, one-half credit will be awarded for the correct reason. For instance, to read the limit off of your graphing calculator would only be worth 1/2 credit. Algebraic reasons for your answer would be worth the remaining 1/2 credit.

\[ \lim_{x \to 7^-} \frac{2x^2 - 17x + 21}{|x - 7|} \]

**Solution:**
\[ \lim_{x \to 7^-} \frac{2x^2 - 17x + 21}{|x - 7|} = \lim_{x \to 7^-} \frac{(2x-3)(x-7)}{|x-7|} = \lim_{x \to 7^-} \frac{(2x-3)(x-7)}{-(x-7)} \] since \( |x-7| = -(x-7) \) when \( x < 7 \). Therefore \( \lim_{x \to 7^-} \frac{2x^2 - 17x + 21}{|x - 7|} = \lim_{x \to 7^-} \frac{2x-3}{(-1)} = \frac{14-3}{(-1)} = -11 \).