Calculator Lab 1 – Preliminaries, Trigonometry

Problem 1

a) Enter the function \( f(x) = \frac{3x - 4}{x^2 + 1} \) into your calculator, set the viewing window to \([-4, 6, -1, 5]\), and graph.

b) Inspect the graph of \( f \), and indicate which points in the interval \([-5, 6]\) can be points in the domain of \( f \). Explain.

c) Use the given formula for \( f \) and your knowledge of functions to describe the largest set of real numbers that can form the domain of \( f \).

d) Use the trace feature of your calculator to compute approximations for the coordinates of the highest point and lowest point of the graph of the function \( f \).

e) Assume that the domain of \( f \) is the interval \([-4, 6]\). Can you use the graph of the function and the information obtained in c) to determine the range of \( f \)? Explain.

f) Use your calculator to construct a table of values for \( f \). Use a starting point of \(-4\), and an increment for \( x \) of 0.1. Scroll through the values in the table to compute approximations for the coordinates of the highest point and the lowest point on the graph. Do these values validate your answer in e)? Give an interval that can be contained in the range of \( f \). Explain.

g) Let \( y = 0 \). Does there exist a value of \( x \) in \([-4, 6]\), such that \( f(x) = y \)? Let \( y = \frac{9}{2} \). Does there exist a value of \( x \) in \([-4, 6]\), such that \( f(x) = y \)? Do the answers to this question validate your answer to f)? Explain.

h) Use algebra to show that \( 0 \leq \frac{3x - 4}{x^2 + 1} \leq \frac{9}{2} \). If the domain of \( f \) is the interval \([-4, 6]\), what is the range of \( f \)? If the domain of \( f \) is the interval \([-\infty, \infty]\), what is the range of \( f \)? Explain.
Problem 2
Let \( f \) be the function described by the graph below.

![Graph of function f]

a) Compute \( f(-5), f(-4), f(-3), f(-2), f(-1), f(0), f(1), f(2), f(3), f(4), f(5) \) if possible. Explain.

b) Does there exist a value of \( x \) such that \( f(x) = -1 \)? Explain.

c) Does there exist a value of \( x \) such that \( f(x) = 1 \)? Explain.

d) Is the function \( f \) a one-to-one function? Does it have an inverse? Explain.

e) Use the information in the graph to define a one-to-one function in the largest possible interval whose values in that interval are identical to the values of the function in the graph. Explain.

f) Use the graph to determine the domain of the function \( f \). Explain.

g) Use the graph to determine the range of \( f \). Explain.

h) Write down a formula for the function. Graph your function to verify that the graph matches the given graph.

Problem 3
The amplitude of an oscillation is half the distance between maximum and minimum values in the oscillation. The period of the oscillation is the time needed for the oscillation to execute a complete cycle. To describe arbitrary amplitudes and periods we use functions of the form \( f(t) = A \sin(Bt) + D \) and \( g(t) = A \cos(Bt) + D \), where \( A \) is the amplitude and \( \frac{2\pi}{B} \) is the period. To represent arbitrary phase differences we shift a graph of the correct amplitude and period by replacing \( t \) with \( t - C \) or \( t + C \). The data below gives the apparent diameter of the Sun's disc (in minutes of arc) as seen from earth at various times during the year. The variation is due to the fact that the Earth's orbit around the Sun is elliptical and we are closer to the Sun at the end of the year. Follow the steps below to find a curve of the form \( g(t) = A \cos(Bt) + D \) that best describes the data; the variable \( t \) represents time, with \( t = 0 \) being January 1. (The approximation you are finding is not computed using regression methods, but rather knowledge of certain functions.)

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>31</th>
<th>59</th>
<th>90</th>
<th>121</th>
<th>152</th>
<th>182</th>
<th>212</th>
<th>243</th>
<th>273</th>
<th>304</th>
<th>334</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(t) )</td>
<td>32.6</td>
<td>32.5</td>
<td>32.3</td>
<td>32.0</td>
<td>31.8</td>
<td>31.6</td>
<td>31.5</td>
<td>31.6</td>
<td>31.7</td>
<td>32.0</td>
<td>32.3</td>
<td>32.5</td>
</tr>
</tbody>
</table>
a) Use your calculator to plot the data points. Set the viewing window to \([0, 400, 31, 33]\). Set tickmarks every 50 days, for the \(x\)-axis, and every 0.2 units for the \(y\)-axis.

b) Study the plot carefully. Use the values in the table and the definition of period of an oscillation to determine an approximate value for \(B\) in the equation \(g(t) = A\cos(Bt) + D\). Note: the year is 365 days long.

c) Study the plot carefully. Use the values in the table and your knowledge of the cosine function to determine an approximate value for \(D\) in the equation \(g(t) = A\cos(Bt) + D\). Plot the equation \(g(t) = \cos(Bt) + D\) using the values that you computed for \(B\) and \(D\). (The function \(g(t) = \cos(Bt) + D\) does not represent the data points accurately, but its graph should be visible in the window. If your graph does not show in the window, you must rethink b) and c).)

d) Study the plot carefully. Use the values in the table and the definition of amplitude of an oscillation to determine an approximate value for \(A\) in the equation \(g(t) = A\cos(Bt) + D\). Plot the equation \(g(t) = A\cos(Bt) + D\) using the values that you computed for \(A\), \(B\) and \(D\). Is this function a good approximation for the data points? Explain.

e) Use your calculator and the trigonometric function you constructed to compute the diameter of the Sun's disc as seen from Earth on February 14, July 10, September 20.

f) Use your knowledge of the sine and cosine functions to determine the phase shift \(C\) needed to approximate the data with the function \(f(t) = A\sin(B(t - C)) + D\). Use your knowledge of the sine and cosine functions to determine the phase shift \(C\) needed to approximate the data with the function \(f(t) = A\sin(B(t + C)) + D\). Explain.

g) Graph the functions that you obtained in f) together with the data points to verify your results.