Problem 1

Make sure that your calculator is graphing in dot style. Let \( f(x) = \begin{cases} 
-x - 3 & \text{if } x < -1 \\
\sqrt{|x|} - 1 & \text{if } -1 < x < 0 \\
(x - 1)^3 & \text{if } 0 \leq x \leq 2 \\
-(x - 2)^2 + \frac{1}{2} & \text{if } x > 2 
\end{cases} \)

a) What is the domain of \( f \)? Compute the values of the function at \( x = -1, 0, 2 \), if possible. Explain.
b) Use your calculator to graph the function \( f \) with viewing window \([-3, 3, -2, 2]\), or a window of your choice that shows the details of the function. Use the graph to determine the range of \( f \).
c) Construct a table of values for \( f \) near \( x = 2 \). Consider values both to the left and to the right of 2. Are the values given in the table consistent with the graph? Explain.
d) From the values observed in the table, give an argument supporting the existence or nonexistence of the limits \( \lim_{x \to 2^+} f(x) \), \( \lim_{x \to 2^-} f(x) \), and \( \lim_{x \to 2} f(x) \).
e) Construct a table of values for \( f \) near \( x = 0 \). Consider values both to the left and to the right of 0. Are the values given in the table consistent with the graph? Explain.
f) From the values observed in the table, give an argument supporting the existence or nonexistence of the limits \( \lim_{x \to 0^+} f(x) \), \( \lim_{x \to 0^-} f(x) \), and \( \lim_{x \to 0} f(x) \).
g) Construct a table of values for \( f \) near \( x = -1 \). Consider values both to the left and to the right of \(-1\). Are the values given in the table consistent with the graph? Explain.
h) From the values observed in the table, give argument supporting the existence or nonexistence of the limits \( \lim_{x \to \pm 1^+} f(x) \), \( \lim_{x \to \pm 1^-} f(x) \), and \( \lim_{x \to \pm 1} f(x) \).
i) Use the results obtained above to find intervals in which the function is continuous. If the function fails to be continuous at some point, indicate which of the defining properties of continuity does not hold.
j) Use the results obtained above to determine if it is possible to (re)define the function in such a way that it becomes continuous at point in which it failed to be continuous.
Problem 2

Make sure your calculator is graphing in dot style. Let \( f(x) = \frac{x^2 + 8x + 13}{x + 3} \).

a) Use your calculator to graph the function \( f \). Use a viewing window that gives sufficient detail.

b) Construct a table of values for \( f \) near \( x = -3 \). Consider values both to the left and to the right of \(-3\). Are the values given in the table consistent with the graph? Explain.

c) From the values observed in the table, give an argument supporting the existence or nonexistence of the limits \( \lim_{x \to -3^+} f(x) \), \( \lim_{x \to -3^-} f(x) \), and \( \lim_{x \to -3} f(x) \). (Use algebra to confirm your answer.)

d) Construct a table of values for \( f \) for large negative values of \( x \). As \( x \) decreases (becomes more and more negative), how are the values of \( f(x) \) changing?

e) From the values observed in the table, give an argument supporting the existence or nonexistence of the limit \( \lim_{x \to -\infty} f(x) \). (Use algebra to confirm your answer.)

f) Enter the function \( g(x) = x + 5 \) into your calculator. Deselect the function \( f \) and construct a table of values for \( g(x) \) for large negative values of \( x \). How do the values in this table compare with the table for \( f(x) \)? Explain.

g) Construct a table of values for \( f \) for large values of \( x \). As \( x \) increases, how are the values of \( f(x) \) changing?

h) From the values observed in the table, give argument supporting the existence or nonexistence of the limit \( \lim_{x \to \infty} f(x) \). (Use algebra to confirm your answer.)

i) Construct a table of values for \( g(x) \) for large values of \( x \). How do the values in this table compare with the table for \( f(x) \)? Explain.

j) Use the information you obtained in d)–i) to give an argument indicating the existence or nonexistence of asymptotes.

k) Graph the functions \( f \) and \( g \) together. Use a viewing window that gives sufficient detail. Is your answer to j) consistent with the graph? (Use algebra to confirm your answer.)