INTERMEDIATE ALGEBRA REVIEW FOR MODULE 11

<u>Solving Linear Equations: Section 1.1</u>

Solving Equations:

- **Clear Fractions**
- **Clear Parentheses**
- **Move the terms with the variable to one side of the equal sign (by adding and subtracting.)**
- **Move the terms without the variable to the other side.**
- **Combine like terms**
- **Divide by the coefficient of x**

1. **Solve each equation for x:**
   a) $3x - 5 = 6x - 10$

   b) $\frac{x}{5} = 3 - \frac{x}{2}$

   c) $3(x - 2) = 5x - 4(x + 1)$

   d) $x - \frac{1}{2} = \frac{2}{3}(x + 3)$

<u>Sets, Interval Notation, and Inequalities: Section 1.4</u>

Interval Notation **describes the covered interval of the number line from left to right..**
Open circles are indicated by parentheses; Closed circles are indicated by brackets and open ends are indicated by $-\infty$ on the left and ∞ on the right.
The steps for solving inequalities are the same as for solving equations *except* if both sides of the equation are multiplied or divided by a negative number, the inequality symbol must be reversed.

2. **Solve the following inequalities: Graph on the number line and express in interval notation.**
   a) $-3x > -6$
   b) $2x - 5 < 3x + 2$
   c) $\frac{x}{2} - 4 \leq 3x + \frac{1}{2}$

<u>Translating word problems into inequalities:</u>

Phrases that translate to inequalities are:

Is more than , Is less than, Is at least, Is at most, Is greater than,
Is greater than or equal to, Is less than or equal to

3. **Write an inequality for each of the following and find the solutions.**
   a) **Three more than twice a number is at least 12. Find the set of values for the number.**

   b) **The formula for calculating Centigrade temperature from Farenheit is $C = \frac{5}{9}(F - 32)$. If a substance melts at temperatures less than 285^0C, what would be the equivalent temperatures in ^{0}F?**

<u>Intersections and Unions: Section 1.5</u>

The union of two sets is the collection of objects that are in either set.(one, the other or both.) In words the union would be an "or."

The Intersection of two sets is the collection of objects that are in both sets. In words, the intersection would be an "and".
In sets of inequalities, the way the inequality is written indicates a union or an intersection:
$3 < x < 5$ indicates all the numbers between 3 and 5, not including 5.
To indicate the or, two inequalities are necessary.
$x < 3$ or $x>5$ indicates all the numbers that are either less than 3 or greater than 5.

4. **Graph each on a number line and write in interval notation.**
   - a) $x \leq -2$ or $x \geq 5$
   - b) $x \leq 5$ or $x \geq -2$
   - c) $-2 < x \leq 5$
   - d) $5 < x \leq -2$

5. **Solve each inequality; graph on the number line and write solution in interval notation.**
   - a) $3x - 5 > 3$ or $2x - 1 \leq 1$
   - b) $-5 < 2x - 1 < 6$

<u>Absolute Value Equations and Inequalities: Section 1.6</u>

Absolute value Equations: **These are solved by creating two equations.** ***The solutions must be checked to be sure they solve the original absolute value equation.*** $|\text{expression}| = b$ **becomes expression = b** ***or*** **expression = - b** ***Absolute value Inequalities*** **are turned into "ands" or "ors" depending on the placement of the absolute value expression relative to the inequality symbol. Each of those inequalities is then solved according to the methods of the previous section.**

$|\text{expression}| < b$ **becomes - b < expression < b**

$|\text{expression}| > b$ **becomes expression < b** ***or*** **expression > -b**

6. **Solve each of the following. Graph the solutions on the number line and express in interval notation.**
   - a) $|2x + 3| = 5$
   - b) $|2x + 3| = -5$
   - c) $|2x + 3| \leq 5$
   - d) $\left|x + \frac{3}{4}\right| > \frac{1}{2}$

<u>Solving Systems of Equations two equations and two unknown: Section3.2,3.3,3.4</u>

- **To solve systems of two equations and two unknowns algebraically:**
  ***Substitution*: Get a letter by itself in one of the equations. Rewrite the other equation replacing the isolated letter using parentheses. Solve the resulting equation and substitute back to get the other variable.**
  ***Elimination*: Multiply or divide both sides of one or both equations to arrange it so one of the variables has the same coefficient in both equations but with opposite signs. Add the equations. One variable will drop out. Solve for the remaining variable and substitute back to get the other variable.**

- **In either algebraic approach, if both variables drop out, the system either has no solution (is inconsistent) or has an infinite number of solutions (is dependent.) If the form of the resulting equation is a=b, the system is inconsistent. If it is a=a, it is dependent. Otherwise it is consistent and independent. (has only one solution.)**

7. **Solve the following system by substitution:**

$$3x - 2y = 8$$
$$x = 2y - 8$$

8. **Solve each of the following systems by elimination (addition):**

a) $3x - y = 2$
$x + y = -6$

b) $2x + 3y = 12$
$4x - 5y = 2$

c) $5x + 7y = 1$
$2x + 3y = 0$

9. **Solve the following system by any method:**

$$\frac{x}{2} - \frac{y}{2} = \frac{5}{4}$$

$$3x + 2y = 10$$

Inconsistent and Dependent Systems:

- **Systems with no solution are called inconsistent systems.**
- **Systems with an infinite number of solutions are called dependent systems**
- **To determine if either situation exists, solve the system by either elimination or substitution. If all the variables drop out, then continue to a final equation. If it is an identity (2=2 or 0=0) then the system is dependent. If it is an equation with no solution, (2=3) then the system is inconsistent.**
- **If the variables do not drop out and you get a single solution for each variable, then the systems are consistent and independent.**
- **On a graph, the lines of an inconsistent system will be parallel. In a dependent system, both equations will result in the same line.**

10. **Determine if each of the following systems are dependent, inconsistent or consistent and independent.**

a) $2x - y = 3$
$y = 2x - 5$

b) $x + y = 2$
$-3x - 3y = -6$

Translating word problems into systems of 2 equations and 2 unknowns.

Label one variable x and the other y. Find two separate pieces of information and translate into two separate equations using x and y labels for unknowns. If necessary, tables can be used to create labels from the original x and y labels.

11. **Write a system of equations that could be used to solve the problem. DO NOT SOLVE THE SYSTEM.**
    a) **The perimeter of a rectangular pool is 180 feet. Twice the length minus five times the width is 110 feet. Find the dimensions of the pool.**

    b) **A video store sells DVD's at $18 apiece and Videotapes at $12 apiece. During one week they took in $11,100 from the sale of 850 items. How many DVD's did they sell?**

    c) **A mixture of 95% ethanol and 80% ethanol is combined to make a 20 liters of a 88% ethanol solution. How many liters of each solution was needed?**

    d) **An investment of $2000 is split into two funds. One pays 5% interest and the other pays 7% interest. If the total interest for one year is $124, find the amount invested at each rate.**

    e) **Two trains leave a station headed in the same direction. One train is traveling 80 mph and the other is traveling 50mph. If the faster train leaves one hour later than the slower one, how far from the station will they be when the slower train is overtaken?**

    f) **A boat can travel 12 miles downstream in the same time it takes to travel 8 miles upstream. If the speed of the current is 2 mph, find the speed the boat could travel in still water.**

<u>Systems of Equations in Three VariablesSection 3.5 and 3.6</u>

Step 1: Pick one pair of equations and use elimination to get rid of one variable.
Step 2: Pick a second pair of equations and eliminate *<u>the same variable</u>.*
Step 3*:* Solve the system of two equations and two unknowns.
Step 4: Substitute back to find the third unknown.
If all variables drop out at any point the system is either inconsistent (if get a=b) or dependent (if get a=a).
If one of the equations is already missing a variable, use that equation and eliminate the missing variable in the remaining pair.

12. **Solve the following system:**

$$\begin{aligned} x + y + z &= 0 \\ 2x + 3y + 2z &= -3 \\ -x + 2y - 3z &= -1 \end{aligned}$$

For word problems with three unknowns, label them x, y, and z. Use three separate pieces of information to write three separate equation, one for each fact relating the variables.

13. **Write a system of equations for each of the following problems: *Do not solve the system.***
    a) **The sum of three numbers is 28. The first number minus three times the second number is 6 more than twice the third. The second is one less than the third. Find the numbers.**

    b) **An individual wants to limit their candy intake to 300 calories a day. A piece of chocolate has 50 calories, a slice of licorice has 20 calories and a lollipop has 38 calories The number of servings of licorice must be three times the number of pieces of chocolate. Twice the number of pieces of chocolate plus six times the licorice will be 4 more than the number of pieces of chocolate. How many servings of each type of dessert can (s)he have?**