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6-1: Finding $R_T$ for Series-Parallel Resistances

- **Overview of Series-Parallel Circuits**
  - A *series-parallel circuit*, or *combination circuit*, combines both series and parallel connections.
  - Most electronic circuits fall into this category. Series-parallel circuits are typically used when different voltage and current values are required from the same voltage source.
    - Series components form a **series string**.
    - Parallel components form a **parallel bank**.
There are three branches in this circuit; sections 1 and 2 are series strings.
Overview of Series-Parallel Circuits

There are three series sections in this circuit; sections 1 and 2 are parallel banks.
To find $R_T$ for a series-parallel circuit, add the series resistances and combine the parallel resistances.

In this diagram, $R_1$ and $R_2$ are in series, and $R_3$ and $R_4$ are in parallel. However, $R_2$ is not in series with the parallel resistances: Resistances in series have the same current, but the current in $R_2$ is equal to the sum of the branch currents $I_3$ and $I_4$.

Fig. 6-1b: Schematic diagram of a series-parallel circuit.
6-1: Finding $R_T$ for Series-Parallel Resistances

- For Fig. 6-1b,
  - The series resistances are:
    \[ 0.5\text{k}\Omega + 0.5\text{k}\Omega = 1\text{k}\Omega \]
  - The parallel resistances are:
    \[ 1\text{k}\Omega \div 2 = 0.5\text{k}\Omega \]
  - The series and parallel values are then added for the value of $R_T$:
    \[ 1\text{k}\Omega + 0.5\text{k}\Omega = 1.5\text{ k}\Omega \]
6-2: Resistance Strings in Parallel

- In this figure, branch 1 has two resistances in series; branch 2 has only one resistance.
- Ohm’s Law can be applied to each branch, using the same rules for the series and parallel components that were discussed in Chapters 4 and 5.

Fig. 6-3a: Series string in parallel with another branch (schematic diagram).
6-2: Resistance Strings in Parallel

- **Series Circuit**
  - Current is the same in all components.
  - \( V \) across each series \( R \) is \( I \times R \).
  - \( V_T = V_1 + V_2 + V_3 + \ldots + \) etc.

- **Parallel Circuit**
  - Voltage is the same across all branches.
  - \( I \) in each branch \( R \) is \( V/R \).
  - \( I_T = I_1 + I_2 + I_3 + \ldots + \) etc.
6-2: Resistance Strings in Parallel

\[ V \]

\( I \) is the same in this section.

\[ V \text{ is the same across each parallel branch.} \]
The current in each branch equals the voltage applied across the branch divided by the branch $R_T$.

The total line current equals the sum of the branch currents for all parallel strings.

The $R_T$ for the entire circuit equals the applied voltage divided by the total line current.

For any resistance in a series string, the $IR$ voltage drop across that resistance equals the string’s current multiplied by the resistance.

The sum of the voltage drops in the series string equals the voltage across the entire string.
In this figure, $R_2$ and $R_3$ are parallel resistances in a bank. The parallel bank is in series with $R_1$.

There may be more than two parallel resistances in a bank, and any number of banks in series.

Ohm’s Law is applied to the series and parallel components as seen previously.

Fig. 6-4a: Parallel bank of $R_2$ and $R_3$ in series with $R_1$ (Original circuit).
To find the total resistance of this type of circuit, combine the parallel resistances in each bank and add the series resistances.

\[
R = \frac{V}{I}
\]

\[
R = \frac{24V}{4A}
\]

\[
6\Omega = \frac{10 \Omega \text{ (of } R_2 + R_3\text{)}}{2 \text{ branches}} + 1\Omega \text{ (} R_1 \text{)}
\]

\[
6\Omega = \frac{24V}{4A}
\]

\[
6\Omega = 5\Omega + 1\Omega
\]
To solve series-parallel (combination) circuits, it is important to know which components are in series with one another and which components are in parallel.

Series components must be in one current path without any branch points.

To find particular values for this type of circuit,

- Reduce and combine the components using the rules for individual series and parallel circuits.
- Reduce the circuit to its simplest possible form.
- Then solve for the needed values using Ohm’s Law.
Example:

- Find all currents and voltages in Fig. 6-5.
  - Step 1: Find \( R_T \).
  - Step 2: Calculate main line current as \( I_T = \frac{V_T}{R_T} \).

Fig. 6-5: Reducing a series-parallel circuit to an equivalent series circuit to find the \( R_T \). (a) Actual circuit. (b) \( R_3 \) and \( R_4 \) in parallel combined for the equivalent \( R_T \).

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Fig. 6-5, cont. (c) $R_T$ and $R_6$ in series added for $R_{13}$. (d) $R_{13}$ and $R_5$ in parallel combined for $R_{18}$. 
Fig. 6-5e: The $R_{18}$, $R_1$, and $R_2$ in series are added for the total resistance of 50$\Omega$ for $R_T$. 

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In solving such circuits, apply the same principles as before:
- Reduce the circuit to its simplest possible form.
- Apply Ohm’s Law.
Example:

In Fig. 6-6, we can find branch currents $I_1$ and $I_{2-3}$, and $I_T$, and voltage drops $V_1$, $V_2$, and $V_3$, without knowing the value of $R_T$. 

Fig. 6-6: Finding all the currents and voltages by calculating the branch currents first.

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Find $I_1$, $I_{2-3}$, and $I_T$.

\[
I_1 = \frac{V}{R} \\
I_1 = \frac{90V}{30\Omega} \quad \text{(parallel branches have the same voltage)} \\
I_1 = 3A
\]
6-5: Analyzing Series-Parallel Circuits with Random Unknowns

\[ I_{2-3} = \frac{V}{R} \]
\[ I_{2-3} = \frac{90V}{20\Omega + 25\Omega} \]
\[ I_{2-3} = \frac{90V}{45\Omega} \]

\[ I_T = I_1 + I_{2-3} \]
\[ I_T = 3A + 2A \]
\[ I_T = 5A \]

\[ I_{2-3} = 2A \]
6-5: Analyzing Series-Parallel Circuits with Random Unknowns

- Find voltage drops $V_1$, $V_2$, and $V_3$:
6-5: Analyzing Series-Parallel Circuits with Random Unknowns

\[ V_1 = V_A \text{ (parallel branches have the same voltage)} \]
\[ V_1 = 90V \]

or

\[ V_1 = I_1R_1 \quad V_2 = I_{2-3}R_2 \quad V_3 = I_{2-3}R_3 \]
\[ V_1 = 3A \times 30\Omega \quad V_2 = 2A(20 \ \Omega) \quad V_3 = 2A(25 \ \Omega) \]
\[ V_1 = 90V \quad V_2 = 40V \quad V_3 = 50V \]

Note: \[ V_2 + V_3 = V_A \]
\[ 40V + 50V = 90V \]
6-5: Analyzing Series-Parallel Circuits with Random Unknowns

\[ R_T = \frac{V_A}{I_T} \]

\[ R_T = \frac{90A}{5A} \]

\[ R_T = 18\Omega \]
6-6: The Wheatstone Bridge

- A Wheatstone bridge is a circuit that is used to determine the value of an unknown resistance.
- The unknown resistor \((R_X)\) is in the same branch as the standard resistor \((R_S)\).

Fig. 6-10: Wheatstone bridge.

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6-6: The Wheatstone Bridge

- Resistors $R_1$ and $R_2$ form the ratio arm; they have very tight resistance tolerances.
- The galvanometer ($M_1$), a sensitive current meter, is connected between the output terminals C and D.
- When $R_1 / R_2 = R_3 / R_4$, the bridge is balanced.
- When the bridge is balanced, the current in $M_1$ is zero.
Using a Wheatstone Bridge to Measure an Unknown Resistance

- \( R_S \) is adjusted for zero current in \( M_1 \).
- When the current in \( M_1 = 0 \text{A} \), the voltage division between \( R_X \) and \( R_S \) is equal to that between \( R_1 \) and \( R_2 \).
Note: When the Wheatstone bridge is balanced, it can be analyzed as two series strings in parallel. Note the following relationship:

\[
\frac{R_X}{R_S} = \frac{R_1}{R_2}
\]

\[
R_X = R_S \times \frac{R_1}{R_2}
\]
In series-parallel circuits, an open or short in one part of the circuit changes the values in the entire circuit.

When troubleshooting series-parallel circuits, combine the techniques used when troubleshooting individual series and parallel circuits.
Effect of a Short in a Series-Parallel Circuit

- The total current and total power increase.

Fig. 6-13: Effect of a short circuit with series-parallel connections. (a) Normal circuit with $S_1$ open. (b) Circuit with short between points A and B when $S_1$ is closed; now $R_2$ and $R_3$ are short-circuited.

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**Effect of a Short in a Series-Parallel Circuit**

The total current increases from 2A with \( S_1 \) open to 10A with \( S_1 \) closed. With \( S_1 \) closed, \( R_2 \) and \( R_3 \) are shorted out.
Effect of an Open in a Series-Parallel Circuit

Fig. 6-14: Effect of an open path in a series-parallel circuit. (a) Normal circuit with $S_2$ closed. (b) Series circuit with $R_1$ and $R_2$ when $S_2$ is open. Now $R_3$ in the open path has no current and zero $IR$ voltage drop.

With $S_2$ open, $R_3$ is effectively removed from the circuit.
6-7: Troubleshooting: Opens and Shorts in Series-Parallel Circuits

- Effect of an Open in a Series-Parallel Circuit

With $S_2$ open the voltage across points $C$ and $D$ equals the voltage across $R_2$, which is 89V. The voltage across $R_3$ is zero.