Average vs. Instantaneous (and Initial) Rates

I. Practice Problem/Example

Q1: Calculate the average rate of decomposition of X between t = 10 s and t = 15 s.

Note: I will assume an uncertainty of about ±0.1 s here (i.e., times are assumed to have 3 SF; I should have written them as 15.0 s, etc; otherwise the final result would have one significant figure, which seems ridiculously imprecise.)

\[
\text{Avg. Rate} = \frac{[X]_{15 s} - [X]_{10 s}}{t - t_i} = \frac{-0.0026 M - 0.0044 M}{15.0 s - 10.0 s} = -0.0015 M/5.0 s = +0.0003 M \cdot s^{-1}
\]

Q2: Estimate the instantaneous rate of decomposition of X at 10 s (by finding the slope of the tangent line drawn)

Note: You could pick any two points on that line to find its slope. For convenience, I usually like to pick the axis intercepts (if they appear on the plot): Here they are the points (20.1 s, 0.0000 M) and (0.0 s, 0.0077 M).

\[
\text{Instantaneous Rate} = \text{slope} = \frac{[X]_{20.1 s} - [X]_{0 s}}{t - t_i} = \frac{-0.0000 M - 0.0077 M}{20.1 s - 0.0 s} = -0.0077 M/20.1 s = +0.00038 M \cdot s^{-1}
\]

- How does this rate compare to the average rate in Q1? It is slightly greater.

Is this as expected? Explain. Yes. Since the rate is decreasing with time, the rate at the start of any time interval should be faster than the average rate over that interval. (And the rate at the end of the interval should be slower than the average rate over that interval.)

Q3: Estimate the initial rate of decomposition of X (Hint: Draw/estimate a tangent line to the curve at t = 0, and then find the slope of that line)

See dotted line drawn as a tangent to the curve at t=0. There will be considerable uncertainty in the slope of this line. As above, I will pick the intercepts for my two points: (11.8 s, 0.0000 M) and (0.0 s, 0.0100 M)

\[
\text{Initial Rate} = \text{slope} = \frac{[X]_{11.8 s} - [X]_{0 s}}{t - t_i} = \frac{-0.0000 M - 0.0100 M}{11.8 s - 0.0 s} = -0.0100 M/11.8 s = +0.00084 M \cdot s^{-1} \quad (\text{*see below})
\]

Given the uncertainty associated with drawing the tangent line, a more reasonable result would be something like \(0.00085 \pm 0.00005 \text{ M} \cdot \text{s}^{-1}\) (2 SF max)